Robust Adaptive Nonlinear Control Law for a General Class of Nonlinear Systems with Operator-Based Hysteresis Models

Qingqing Wang * **Chun-Yi Su** * **Ying Feng** * **Shuzhi Sam Ge** **

* *Department of Mechanical Engineering, Concordia University, Montreal, Quebec, H3G 1M8, Canada.*
** *Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576*

Abstract: For the nonlinear systems preceded by smart actuators which exhibit hysteresis nonlinearities, it is a challenge to mitigate effects of the hysteresis. By utilizing an operator-based Prandtl-Ishlinskii model and a neural network approximator, a robust adaptive control scheme is developed for a general class of continuous-time nonlinear dynamic systems with unknown hysteresis nonlinearities. The boundedness of the closed-loop system is achieved and the tracking error converges to a set of adjustable neighborhood of zero independent of initial conditions. The effectiveness of the proposed control approach is demonstrated through a simulation example.

Keywords: Hysteresis, robust adaptive control, neural network

1. INTRODUCTION

Smart-material based actuators, such as piezoceramics and shape memory alley, are becoming increasing important in the positioning technology Tao et al. (1995) Banks et al. (2000). However, the hysteresis behavior always exists in such actuators. When a system is actuated by these actuators, the system performance will be degraded due to the hysteresis nonlinearities. The difficulty of the controller design is due to the non-smooth and multi-valued nature of the hysteresis nonlinearities. It is typically challenging in developing a general frame for control of a system in the presence of unknown hysteresis.

To address such a challenge, it necessitates the development of mathematical models to characterize hysteresis nonlinearities sufficiently accurate Gorbet (1997) Hughes et al. (1997) Tan et al. (2001). Such models should also be amenable to control design for nonlinearity compensation and be efficient to use in real-time applications krejci et al. (2001). Several control models have been widely accepted and found applications in different areas Banks et al. (2000) Brokake et al. (1996) Krasnoskl'kii et al. (1983) Macki et al. (1993) Mayergoyz (1991) and Visintin (1994). Therein, the operator-based models such as the Prandtl-Ishlinskii model has received much attention in modeling the smart actuators. This hysteresis model consists of play operator and stop operator and can describe multiform hysteresis by choosing different density function parameters Brokake et al. (1996).

In Su et al. (2005) and Wang et al. (2006), the Prandtl-Ishlinskii model has already been fused with variable structure control and robust control approaches to mitigate the effects of unknown hysteresis without the inverse hysteresis construction, where the theoretical stability analysis were conducted. For a general class of continuous-time nonlinear dynamic systems with parametric uncertainties and unknown nonlinear functions, systematic controller design procedures are now available, see Ge et al. (1999) Loh et al. (1999) Yao et al. (2001) and the references therein. When this general class of nonlinear dynamic systems Ge et al. (1999) is preceded by actuators with unknown hysteresis represented by Prandtl-Ishlinskii model, a robust control approach is proposed in this paper. Using a neural networks approximator, the proposed control law ensures the boundedness of all the signals in the closed-loop system and the tracking error converges to a set of adjustable neighborhood of zero independent of initial conditions. From the results of the simulations, the undesirable inaccuracies and oscillations due to unknown hysteresis can be mitigated effectively.

2. HYSTERESIS MODELS

In this paper the hysteresis nonlinearity is represented by the Prandtl-Ishlinskii model. Using this model, it is possible to integrate the hysteresis model with available adaptive control structures to mitigate the effects of the hysteresis.

2.1 Play Operators

Before giving the Prandtl-Ishlinskii model, we list below some basic well-known hysteresis operator. A detailed discussion on this subject can be found in the monographs Brokake et al. (1996) Krasnoskl'kii et al. (1983) Visintin (1994). For $r \geq 0$ and a general initial value $w_{-1} \in R$, the play operator $F_r[.;w_{-1}] : C_m[0,t_E] \times R \to C_m[0,t_E]$ with threshold r is then inductively defined by

$$\begin{aligned} &F_r[v;w_{-1}](0) = f_r(v(0), w_{-1}), \\ &F_r[v;w_{-1}](t) = f_r(v(t), F_r[v;w_{-1}](t_i)), \\ &\quad for \quad t_i < t \leq t_{i+1} \quad and \quad 0 \leq i \leq N-1, \end{aligned} \tag{1}$$

with

$$f_r(v,w) = \max(v-r, \min(v+r, w)). \qquad (2)$$

where $0 = t_0 < t_1 < \cdots < t_N = t_E$ is a partition of $[0, t_E]$, such that the function v is monotone on each of the sub-intervals $[t_i, t_{i+1}]$.

In the sequel, we will simply write $F_r[v]$ instead of $F_r[v; w_{-1}]$ so long as doing so does not affect the proof. Due to the natural of play and stop operators, above discussions are defined on the space $C_m[0, t_E]$ of continuous and piecewise monotone functions; however, they can also be extended to the space $C[0, t_E]$ of continuous functions.

2.2 Prandtl-Ishlinskii Model

We are ready to introduce the Prandtl-Ishlinskii model defined by the play hysteresis operators. the Prandtl-Ishlinskii model was introduced through the play operator as follows:

$$w(t) = p_0 v(t) - \int_0^R p(r) F_r[v](t) dr, \qquad (3)$$

where $p(r)$ is a given density function, satisfying $p(r) \geq 0$ with $\int^\infty rp(r)dr < \infty$, and is expected to be identified from experimental data. With the defined density function, this operator maps $C[t_0, \infty)$ into $C[t_0, \infty)$,i.e., Lipschitz continuous inputs will yield Lipschitz continuous outputs Krasnoskl'kii et al. (1983). $p_0 = \int_0^R p(r)dr$ is constant which depends on the density function. Since the density function $p(r)$ vanishes for large values of r, the choice of $R = \infty$ as the upper limit of integration in the literature is just a matter of convenience Brokake et al. (1996).

where It should be noted that Equation (3) decomposes the hysteresis behavior into two terms. The first term describes the linear reversible part and the second term describes the nonlinear hysteretic part. This decomposition is crucial since it facilitates the utilization of the currently available robust adaptive control techniques for the controller design.

3. PROBLEM STATEMENT

We consider a SISO nonlinear system with the hysteresis presented as an input

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \qquad i = 1, 2, \cdots, n-1 \\ \dot{x}_n &= a(x) + b(x) w(t) + d_e(t) \\ y &= x_1 \end{aligned} \qquad (4)$$

where $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the system state; $a(x)$ and $b(x)$ are unknown smooth functions; $d_e(t)$ represents the system uncertainties such as the external disturbances and modelling errors bounded by a known constant $d_0 > 0$, i.e., $|d_e(t)| \leq d_0$. $w(t)$ is the hysteresis operator given in

$$w(t) = p_0 v(t) - d[v](t) \qquad (5)$$

$$d[v](t) = \int_0^R p(r) F_r[v](t) dr \qquad (6)$$

where $p_0 = \int_0^R p(r)dr$. For convenience, $F_r[v, \psi_{-1}]$ is denoted by $F_r[v]$ for any given hysteresis initial state $\psi_{-1} \in \Psi$. By using this hysteresis model, system (4) becomes

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \qquad i = 1, 2, \cdots, n-1 \\ \dot{x}_n &= a(x) + b(x) p_0 v(t) - b(x) d[v](t) + d_e(t) \\ y &= x_1 \end{aligned} \qquad (7)$$

In this paper, we study the adaptive control problem of the physical plants operating in bounded regions, the state variable belongs to a compact set $\Omega_x \subset R^n$. The objective is to design a stable control law $v(t)$, to force the state vector $x = [x_1, x_2, \cdots, x_n]^T \in \Omega_x$ to follow a specified desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, ..., x_d^{(n-1)}]^T$ as close as possible.

For the considered systems the following assumptions are made:

Assumption 1: The sign of $b(x)$ is known and there exists a constant $b_0 > 0$, $b_0 < |b(x)|$, $\forall x \in \Omega_x$. Since the sign of $b(x)$ is known and $b(x)$ is not equal to zero, we may assume that $b(x) > 0$.

Assumption 2: There exists a smooth function $\bar{b}(x)$ such that $|b(x)| \leq \bar{b}(x)$ and $b(x)/\bar{b}(x)$ is independent of the state x_n, $\forall x \in \Omega_x \subset R^n$.

Assumption 3: The desired trajectory $\mathbf{x}_d \in C^{(n)}(R)$ is available and $\mathbf{x}_d \in \Omega_d \subset R^n$ with Ω_d a compact set.

Assumption 4: There exist a known constant $p_{0min} > 0$ and a known function $p_{max}(r)$, such that $p_0 > p_{0min}$, and $p(r) \leq p_{max}(r)$ for all $r \in [0, R]$.

Remark: Assumption 1 and 3 are generally adopted for the design of tracking controller. As mentioned in Ge et al. (1999), Assumption 2 imposes an additional restriction on the class of systems. However, many physical systems possess such a property, such as, pendulum plants, magnetic levitation systems, and single link robots with flexible joints. As for Assumption 4, based on the properties of the density function $p(r)$, it is reasonable to set an upper bound p_{max} for $p(r)$. Here $p_{0min} > 0$ must be satisfied, otherwise $p_0 = 0$ implies $w(t) = 0$.

To simplify the notification, let $g(x) = b(x)p_0/\bar{b}(x)p_{0max}$, where $p_{0max} = \int_0^R p_{max}(r)dr$, from Assumptions 1-3, $g(x)$ is independent of x_n and $0 < g(x) \leq 1$.

Define the tracking error vector $\tilde{\mathbf{x}}$ as $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d$, and a filtered tracking error as

$$s(t) = (\frac{d}{dt} + \lambda)^{(n-1)} \tilde{x}_1(t), \qquad \lambda > 0 \qquad (8)$$

$s(t)$ can be rewritten as $s(t) = [\Lambda^T 1]\tilde{\mathbf{x}}(t)$ with $\Lambda^T = [\lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, ..., (n-1)\lambda]$.

It has been shown in Slotine et al. (1991) that the definition given in (8) has the following properties: (i) the equation $s(t) = 0$ defines a time-varying hyperplane in R^n on which the tracking error vector $\tilde{\mathbf{x}}(t)$ decays exponentially to zero, (ii) if $\tilde{\mathbf{x}}(0) = 0$ and $|s(t)| \leq \epsilon$, where ϵ is a constant, then $\tilde{\mathbf{x}}(t) \in \Omega_\epsilon \triangleq \{\tilde{\mathbf{x}}(t) \mid |\tilde{\mathbf{x}}_i| \leq 2^{i-1}\lambda^{i-n}\epsilon, i = 1, ..., n\}$ for $\forall t \geq 0$, and (iii) if $\tilde{\mathbf{x}}(0) \neq 0$ and $|s(t)| \leq \epsilon$, then $\tilde{\mathbf{x}}(t)$ will converge to Ω_ϵ within a time-constant $(n-1)/\lambda$.

4. CONTROLLER DESIGN

In this section, we first assume that nonlinear functions $a(x)$, $b(x)$ are known exactly, hysteresis weight function

$p(r)$ is available, and the system uncertainty $d_e(t) = 0$. Notice that the Prandtl-Ishlinskii model (5) decomposes the hysteresis behavior into two terms. The linear reversible component $p_0 v(t)$ and the nonlinear hysteretic component $d[v](t)$. If $d[v](t) = 0$, the system input is $w(t) = p_0 v(t)$, there exists an ideal feedback control v^* as suggested in Ge et al. (1999). Under this control the state vector $\mathbf{x}$ will follows the desired trajectory $\mathbf{x_d}$ asymptotically.

Consider the state feedback control

$$v^*(t) = \frac{1}{\bar{b}(x)p_{0max}} v_n^*(t) \tag{9}$$

with

$$v_n^*(t) = -\frac{1}{g(x)}[a(x)+\mu] - s[\frac{1}{\delta g(x)} + \frac{1}{\delta g^2(x)} - \frac{\dot{g}(x)}{2g^2(x)}],$$

where $\delta > 0$ is a constant and $\mu = (0, \Lambda^T)\tilde{\mathbf{x}} - y_d^{(n)}$

By definition (8), the time derivative of s with the input $v^*(t)$ for the system (7) can be written as

$$\dot{s}(t) = -[\frac{1}{\delta} + \frac{1}{\delta g(x)} - \frac{\dot{g}(x)}{2g(x)}]s. \tag{10}$$

Define a Lyapunov function candidate $V_1 = \frac{1}{2g(x)}s^2$, the time derivative of V_1 along (10) equals

$$\dot{V}_1(t) = -\frac{2}{\delta}[1 + \frac{1}{g(x)}]\frac{1}{2g(x)}s^2. \tag{11}$$

Since $0 < g(x) \leq 1$, it follows that

$$\dot{V}_1(t) \leq -\frac{4}{\delta}V_1. \tag{12}$$

the solution of the above inequality satisfies

$$V_1(t) \leq e^{-\frac{4}{\delta}(t-t_0)}V_1(t_0), \quad \forall \ t \geq t_0 \tag{13}$$

$|b(x)| \geq b_0 > 0$, $\lim_{t\to\infty} V_1(t) = 0$ implies $\lim_{t\to\infty} s = 0$. Furthermore, using properties of s, $\lim_{t\to\infty} \|\tilde{\mathbf{x}}\| = 0$.

We have proved that when functions $a(x)$ and $b(x)$ are known, the hysteresis weight function $p(r)$ is available, using the control input v^* defined in (9), the tracking error vector $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d$ converges asymptotically to zero if $d[v](t) = 0$ and the system uncertainty $d_e(t) = 0$.

When $a(x)$, $b(x)$, and $p(r)$ are unknown, the controller v^* given in (9) cannot be implemented. A reasonable idea is to use estimated v to approximate $v^*(t)$. From previous discussion, v_n^* exists. Under Assumptions 1-2, $a(x)$ and $b(x)$ are continuous functions of x, v_n^* is continuous respect to $x(t)$ and $\mathbf{x}_d$. It has been assumed that $\mathbf{x}_d$ is continuous on the compact set Ω_d and $x(t)$ takes values in compact set Ω_x, all conditions for the Universal Approximation theorem are satisfied. Therefore, function approximation methods such as neural networks or fuzzy systems can be applied. In what follows, neural networks will be used to approximate v_n^* as shown in Ge et al. (1999).

Now let $z = (x^T, s, s/\delta, \mu)^T$, z belongs to a compact set $\Omega_z = \{(x^T, s, s/\delta, \mu)|x \in \Omega_x, x_d \in \Omega_d\}$, v_n^* is function of z. As mentioned in Ge et al. (1999), s and s/δ are in different scales when a small δ is chosen. Feeding the neural networks with both s and s/δ will improve the approximation accuracy. For any arbitrary constant ϵ_0, there exists an integer l^*, such that for all $l \geq l^*$, the following approximation holds:

$$v_n^*(t) = \theta^{*T}\Phi(z) + \epsilon_l \qquad \forall z \in \Omega_z \tag{14}$$

where l is the number of neural networks node, $\Phi(z) \in R^l$ is the basis function vector, the approximation error ϵ_l satisfies $|\epsilon_l| \leq \epsilon_0$. θ^* is the ideal weight defined as

$$\theta^* = arg\ min_{\theta\in R^l}\{sup_{z\in\Omega_z}|\theta^T\Phi(z) - v_n^*(t)|\} \tag{15}$$

Now, the unknown nonlinearity problem transformed into a problem to estimate the idea parameter vector θ^*. Let $\hat{\theta}$ be an estimate of the ideal neural networks weight θ^*, and the controller $v_n(t)$ is chosen as

$$v_n(t) = \hat{\theta}^T\Phi(z) \tag{16}$$

with the adaptation law

$$\dot{\hat{\theta}} = -\Gamma[\Phi(z)s + \sigma\hat{\theta}] \tag{17}$$

where $\Gamma, \sigma > 0$ are adaptive gains.

In order to cancel the effect caused by the term $d[v](t)$, we notice that $d[v](t)$ is determined by the weight function $p(r)$, $p(r)$ is not a function of t. So it can be considered as a parameter for each fixed $r \in [0, R]$ and adjusted by adaptation law. Let $\hat{p}(t, r)$ be the estimate of $p(r)$ at any $r \in [0, R]$, define

$$v_h(t) = \int_0^R \frac{\hat{p}(t,r)}{p_{0min}}|F_r[v](t)|dr sign(s) \tag{18}$$

with adaptation law

$$\frac{\partial}{\partial t}\hat{p}(t,r) = -\gamma\eta\hat{p}(t,r) + \eta\frac{\bar{b}(x)p_{0max}}{p_{0min}}|F_r[v](t)||s| \tag{19}$$

where $\gamma > 0$ and $\eta > 0$ are adaptive parameters.

Now, the adaptive controller is defined as following

$$v(t) = \frac{1}{\bar{b}(x)p_{0max}}v_n(t) + v_h(t) \tag{20}$$

where v_n and v_h are given by (16) and (18). Substituting $v(t)$ into system (7), the time derivative of s can be rewritten as

$$\dot{s}(t) = [a(x)+\mu] + g(x)v_n + b(x)p_0 v_h(t) \tag{21}$$
$$-b(x)\int_0^R p(r)F_r[v](t)dr + d_e(t)$$

To establish global boundedness, let

$$\tilde{\theta} = \hat{\theta} - \theta^*, \tag{22}$$
$$\tilde{p}(t,r) = \hat{p}(t,r) - p(r), \quad \forall\ r \in [0, R] \tag{23}$$

we choose the Lyapunov function candidate as

$$V(t) = \frac{1}{2g(x)}s^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr \tag{24}$$

then, the time derivative of V is

$$\dot{V}(t) = -[\frac{1}{\delta g(x)} + \frac{1}{\delta g^2(x)}]s^2 - \epsilon_l s + \frac{d_e(t)}{g(x)}s \tag{25}$$
$$+\tilde{\theta}^T[\Phi(z)s + \Gamma^{-1}\dot{\hat{\theta}}] + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\hat{p}(t,r)dr$$
$$+\bar{b}(x)p_{0max}s[v_h(t) - \int_0^R \frac{p(r)}{p_0}F_r[v](t)dr] \tag{26}$$

Utilizing adaptation law (17), we can obtain

$$\tilde{\theta}^T[\Phi(z)s + \Gamma^{-1}\dot{\hat{\theta}}] = -\sigma\tilde{\theta}^T\hat{\theta} \tag{27}$$

To simplify the last three terms in (26), from definition (18), we have

$$
\begin{aligned}
&\bar{b}(x)p_{0max}s[v_h(t) - \frac{1}{p_0}\int_0^R p(r)F_r[v](t)dr] \\
&+\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\
\leq &\frac{\bar{b}(x)p_{0max}|s|}{p_{0min}}[-\int_0^R (\hat{p}(t,r)-p(r))|F_r[v](t)|dr] \\
&+\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr
\end{aligned}
$$

Noticing that $\tilde{p}(t,r) = \hat{p}(t,r) - p(r)$, and substituting (19) into the above equation, we have

$$
\begin{aligned}
&-\frac{\bar{b}(x)p_{0max}|s|}{p_{0min}}\int_0^R (\hat{p}(t,r)-p(r))|F_r[v](t)|dr \\
&+\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\
\leq &-\gamma\int_0^R \tilde{p}(t,r)\hat{p}(t,r)dr
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\dot{V}(t) \leq &-[\frac{1}{\delta g(x)} + \frac{1}{\delta g^2(x)}]s^2 - \epsilon_l s \\
&+\frac{d_e(t)}{g(x)}s - \sigma\tilde{\theta}^T\hat{\theta} - \gamma\int_0^R \tilde{p}(t,r)\hat{p}(t,r)dr
\end{aligned} \tag{28}
$$

Furthermore, using the following inequalities

$$
-\sigma\tilde{\theta}^T\hat{\theta} \leq -\frac{\sigma}{2}\|\tilde{\theta}\|^2 + \frac{\sigma}{2}\|\theta^*\|^2 \tag{29}
$$

$$
|\epsilon_l s| \leq \sqrt{2}|\epsilon_l s| \leq \frac{1}{\delta g(x)}s^2 + \frac{\delta g(x)}{2}\epsilon_l^2 \tag{30}
$$

$$
|\frac{d_e(t)}{g(x)}s| \leq \frac{1}{2\delta g^2(x)}s^2 + \frac{\delta}{2}d_e^2(t) \tag{31}
$$

$$
-\tilde{p}(t,r)\hat{p}(t,r) \leq -\frac{1}{2}\tilde{p}^2(t,r) + \frac{1}{2}p^2(r) \tag{32}
$$

and noticing that $0 < g(x) \leq 1$, $|\epsilon_l| \leq \epsilon_0$, and $|d_e(t)| \leq d_0$,

$$
\begin{aligned}
\dot{V}(t) \leq &-\frac{1}{2\delta g(x)}s^2 - \frac{\sigma}{2}\|\tilde{\theta}\|^2 - \frac{\gamma}{2}\int_0^R \tilde{p}^2(t,r)dr + \frac{\delta}{2}\epsilon_0^2 \\
&+\frac{\delta}{2}d_0^2 + \frac{\sigma}{2}\|\theta^*\|^2 + \frac{\gamma}{2}\int_0^R p^2(r)dr
\end{aligned} \tag{33}
$$

let

$$
\tau = min(\frac{1}{\delta}, \frac{\sigma}{\lambda_{max}}, \gamma\eta) \tag{34}
$$

where λ_{max} is the largest eigenvalue of Γ^{-1}, the above inequality satisfies

$$
\dot{V}(t) \leq -\tau V + \frac{c}{2} \tag{35}
$$

with

$$
c = \delta\epsilon_0^2 + \delta d_0^2 + \sigma\|\theta^*\|^2 + \gamma\int_0^R p^2(r)dr \tag{36}
$$

we have $V(t) \leq e^{-\tau(t-t_0)}V(t_0) + \frac{c}{2\tau}$

From the definition of V, we conclude that s, $\tilde{\theta}$, and $\tilde{p}$ are bounded. Especially,

$$
|s(t)| \leq \sqrt{2V(t)} \leq \sqrt{2V(t_0)}e^{-\frac{\tau(t-t_0)}{2}} + \sqrt{\frac{c}{\tau}}
$$

Noticing that the bound for the filtered tracking error in (37) is a function of t and depends on the initial value $V(t_0)$. Using the same method as in Slotine et al. (1991), we can prove that the tracking error vector $\tilde{\mathbf{x}}$ converges to a set, which is not depend on the initial condition $V(t_0)$. Let $p = d/dt$ be the Laplace operator,

$$
\begin{aligned}
y_1(p) &= \frac{1}{p+\lambda}s(p) \\
y_i(p) &= \frac{1}{p+\lambda}y_{i-1}, \qquad i = 1, 2, \cdots, n-1
\end{aligned} \tag{37}
$$

from (37), $y_1(t)$ is bounded by

$$
\begin{aligned}
|y_1(t)| &\leq \int_{t_0}^t e^{-\lambda(t-\alpha)}|s(\alpha)|d\alpha \\
&\leq \begin{cases} \frac{1}{\lambda}\sqrt{\frac{c}{\tau}} + \sqrt{2V(t_0)}e^{-\lambda(t-t_0)}(t-t_0) \\ \text{if } \lambda = \tau/2; \\ \frac{1}{\lambda}\sqrt{\frac{c}{\tau}} + \frac{2\sqrt{2V(t_0)}}{2\lambda-\tau}\{e^{-\frac{\tau}{2}(t-t_0)} - e^{-\lambda(t-t_0)}\} \\ \text{otherwise} \end{cases}
\end{aligned}
$$

by integrating inequality $|y_i(t)| \leq \int_{t_0}^t e^{-\lambda(t-\alpha)}|y_{i-1}(t)|d\alpha$ from $i = 2$ to $i = n-1$, for $n \geq 3$, we have

$$
|y_{n-1}(t)| \leq \int_{t_0}^t e^{-\lambda(t-\alpha)}|y_{n-2}(t)|d\alpha
$$

$$
\leq \begin{cases} \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}} + \frac{\sqrt{2V(t_0)}}{(n-1)!}(t-t_0)^{n-1}e^{-\lambda(t-t_0)} \\ \text{if } \lambda = \tau/2; \\ \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}} + \sqrt{2V(t_0)}\{(\frac{2}{2\lambda-\tau})^{n-1}e^{-\frac{\tau}{2}(t-t_0)} \\ -\sum_{i=1}^{n-1} (\frac{2}{2\lambda-\tau})^{n-i}\frac{(t-t_0)^{i-1}}{(i-1)!}e^{-\lambda(t-t_0)}\} \\ \text{otherwise} \end{cases} \tag{38}
$$

since $\tilde{x}_1(t) = y_{n-1}(t)$, $\tilde{x}_1(t)$ satisfies the above inequality, and $\lim_{t\to\infty}\tilde{x}_1(t) = \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}}$, where c and τ are existing constants given in (34) and (36). The upper bounds for $|\tilde{x}_1(t)|$, $\forall t \geq t_0$ are also given as

$$
|\tilde{x}_1(t)| \leq \begin{cases} \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}} + \frac{\sqrt{2V(t_0)}}{(n-1)!}(\frac{n-1}{\lambda})^{n-1}e^{-(n-1)} \\ \text{if } \lambda = \tau/2; \\ \\ \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}} + \sqrt{2V(t_0)}\{(\frac{2}{2\lambda-\tau})^{n-1}e^{-\frac{\tau}{2\lambda}(i-1)} \\ -\sum_{i=1}^{n-1} (\frac{2}{2\lambda-\tau})^{n-i}\frac{(i-1)^{i-1}}{(i-1)!\lambda^{i-1}}e^{-(i-1)}\} \\ \text{otherwise} \end{cases}
$$

they are reached at $t = \frac{n-1}{\lambda} + t_0$ for $\lambda = \tau/2$ and $t = \frac{i-1}{\lambda} + t_0$ otherwise.

Similarly, for $\tilde{x}_i(t), i = 2, \cdots, n-1$, let

$$\begin{aligned} y_1(p) &= \frac{1}{p+\lambda} s(p) \\ y_j(p) &= \frac{1}{p+\lambda} y_{j-1}(p), \qquad j = 1, 2, \cdots, n-i-1 \\ z_1(p) &= y_{n-i-1}(p) \\ z_j(p) &= \frac{s}{s+\lambda} z_{j-1}(p), \qquad j = 2, \cdots, i \end{aligned} \tag{39}$$

since $\tilde{x}_i(t) = z_i(t)$, using previous results we can prove that

$$\lim_{t\to\infty} \tilde{x}_i(t) = \lim_{t\to\infty} z_i(t) = 2^{i-1}\lambda^{i-n}\sqrt{c/\tau} \tag{40}$$

Theorem: Consider nonlinear system (4) with the hysteresis as an input represented by the Prandtl-Ishlinskii model satisfying Assumptions 1)-4), if the robust adaptive controller is specified by (20) with adaptation laws (17) and (19), then for any bounded initial conditions, all closed-loop signals are bounded and the state vector $x(t)$ converges to

$$\Omega_\epsilon = \{x(t) |\, |\tilde{x}_i| \le 2^{i-1}\lambda^{i-n}\sqrt{c/\tau}, i = 1, ..., n\}$$

where τ and c are constants given in (34) and (36).

5. SIMULATION STUDIES

To illustrate the proposed robust adaptive control algorithm, we use the same example given in Ge et al. (1999). The plant dynamics can be expressed in (4).

If the parameters satisfy $l(\phi) = l_0 + l_1 cos(\phi)$, $l_1/l_0 = 0.5$, $g/l_0 = 10$ and $ml_0^2 = 1$, we have

$$\begin{aligned} a(x) &= \frac{0.5 sinx_1(1 + 0.5cosx_1)x_2^2 - 10sinx_1(1 + cosx_1)}{0.25(2 + cosx_1)^2} \\ b(x) &= \frac{1}{0.25(2 + cosx_1)^2} \\ d_e(t) &= d_1(t)cosx_1 \qquad with \qquad d_1(t) = cos(3t) \end{aligned}$$

where $x = [x_1, x_2]^T = [\phi, \dot{\phi}]^T$. The state variables belong to the compact set

$$\Omega_x = \{(x_1, x_2) |\, |x_1| \le \pi/2,\ |x_2| \le 4\pi\} \tag{41}$$

It can be checked that $4/9 \le b(x) \le 1$ for all $x \in \Omega_x$, Assumption 1 and 2 are satisfied. We set $\bar{b}(x) = 1$. The reference signal is given as $y_d = sin(2t)$. The initial states are assumed to be $[x_1(0), x_2(0)]^T = [0, 0]^T$ and $\lambda = 5$.

$w(t)$ is the output of the hysteresis operator expressed by the Prandtl-Ishlinskii model where $p(r) = \alpha e^{-\beta(r-0.5)^2}$ for $r \in [0, 100]$, with $\alpha = 0.45, \beta = 0.009$. $p_0 = 5.379$, $p_{0max} = 6.379$ and $p_{0min} = 4.379$. We also assume that the hysteresis internal state was $\psi(r) = 0.15$ for $r \in [0, 100]$ before $v(0)$ was applied. For the calculation of $\hat{B}(t)$, we replace the integration by the summation $\sum_0^N$. In the simulation, we choose $N = 4000$. The sampling time is 0.005. To avoid the vibration caused by the discontinues sign function, we use saturate function $sat(s/\epsilon) = s/\epsilon$ instead of $sign(s)$. The proof is still true except in a small neighborhood of $(-\epsilon, \epsilon])$. Here we take $\epsilon = 0.1$.

In this example, a two-order neural network with the NN node number $l = 20$ is selected as

$$f_n(\theta, z) = \hat{\theta}^T \Phi(z) \qquad \forall z \in \Omega_z \tag{42}$$

where $z = [x_1, x_2, s, s/\delta, \mu]$ and θ is the vector of weight parameters. The basis function is $\Phi(z) = [\phi_1(z), \phi_2(z), \ldots, \phi_{20}(z)]^T$. If we choose $\phi(z_j)$ as a hyperbolic tangent function

$$\phi(z_j) = \frac{e^{z_j} - e^{-z_j}}{e^{z_j} + e^{-z_j}}, \qquad j = 1, \ldots, 5 \tag{43}$$

then $\phi_i(z)$ are the possible combinations of $\phi(z_j)^{d_{i,1}}$ $\phi(z_k)^{d_{i,2}}$, where $d_{i,1}$ and $d_{i,2}$ are non-negative integers, for $j, k = 1, \ldots, 5$.

Under the basis function defined as above, for the neural network initial weight vector $\theta = 0$, we set the neural network adaptation gains as $\Gamma = diag\{35\}$, $\sigma = 0.1$, and $\delta = 0.05$. For the adaptation laws of hysteresis, we choose adaption parameters $\gamma = 0.11$ and $\eta = 0.15$. simulation results are shown in Fig. 1 to Fig. 4. Figs. 1-2 show the tracking errors of the system states for the desired trajectory. It can be seen that with the term $v_h(t) \ne 0$ designed to reduce the effects of hysteresis, system state errors $\tilde{x}_1 = x_1 - x_d$ and $\tilde{x}_2 = x_2 - \dot{x_d}$ converge to a small region after 2 seconds. In Fig. 3, the norm of the estimated neural network weight parameters $\|\hat{\theta}\|$ is bounded. The control input $v(t)$ designed to reduce the hysteresis effect are given in Fig. 4. To illustrate the effectiveness of the proposed control scheme, the simulation has also been conducted without controlling the effects of hysteresis, which is implemented by setting $v_h(t) = 0$ in the controller $v(t)$. This implies that the control compensation for the hysteresis nonlinearity is ignored. The results are presented in the figures with dashed lines. We can see that the proposed robust controller clearly demonstrates excellent tracking performance and the developed control algorithm can overcome the effects of the hysteresis.

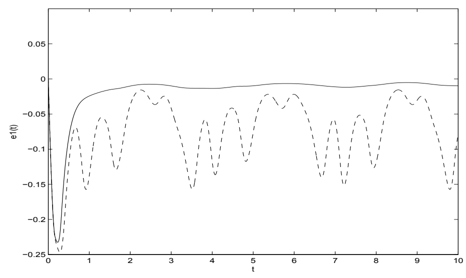

Fig. 1. Tracking errors of the system state for the desired trajectory $\tilde{x}_1 = x_1 - x_d$ in the time spans of 10 seconds. Where solid lines represent the result with control term $v_h \ne 0$ and dashed lines with $v_h = 0$.

6. CONCLUSION

In this paper, a robust adaptive control architecture is proposed for a general class of continuous-time nonlinear dynamic systems preceded by a hysteresis nonlinearity with bounded disturbances. By utilizing the operator-based Prandtl-Ishlinskii model and neural networks approximator, a robust adaptive control scheme is developed. The control law ensures that all the close-loop system

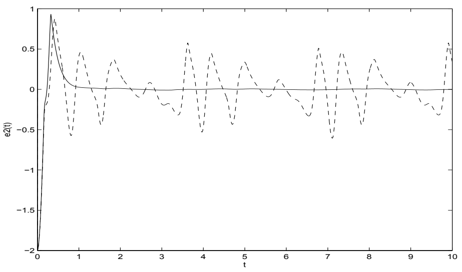

Fig. 2. Tracking errors of the system state for the desired trajectory $\tilde{x}_2 = x_2 - \dot{x_d}$ in the time spans of 10 seconds. Where solid lines represent the result with control term $v_h \neq 0$ and dashed lines with $v_h = 0$.

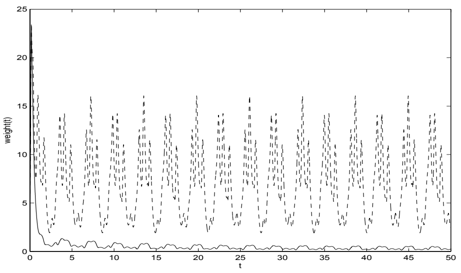

Fig. 3. Neural network weight $\|\hat{\theta}\|$. Where solid line represents the result with control term $v_h \neq 0$ and dashed line with $v_h = 0$.

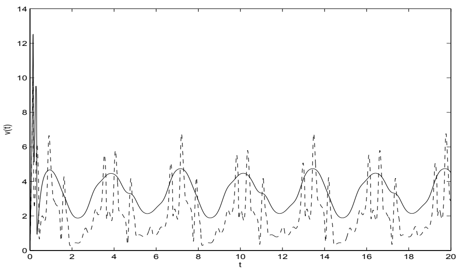

Fig. 4. The control input $v(t)$. Where solid line represents the result with control term $v_h \neq 0$ and dashed line with $v_h = 0$.

signals are bounded, and the tracking error converges to a adjustable neighborhood of zero which is independent of the initial conditions. Simulation results have confirmed the effectiveness of the proposed control approach.

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