Method of Inequality-Based Multiobjective Genetic Algorithm for Optimizing Cart-Double-Pendulum-System

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Abstract: This article presents a multiobjective genetic algorithm to tracking the optimal parameterization problem of the controller concerning the swinging-up and handstand-control of the general cart-double-pendulum system (CDPS). The design based on the Human-Simulated Intelligent Control (HSIC) theory is required to meet various criteria according to the expected specifications. The proposed algorithm extends from the original method of inequality-based multiobjective genetic algorithm (MMGA) and can efficiently maintain the Pareto set of the CDPS optimal parameters in the evolutionary population.

1. INTRODUCTION

The physical cart-double-pendulum system (CDPS) contains two-link manipulators mounted on a cart where pendulums are passive and only the cart is actuated as shown in Fig. 1. CDPS is a typical nonlinear and under-actuated system with some characteristics of complex systems. According to different preliminary states, the handstand pendulum control problem contains balance and swing-up phases.

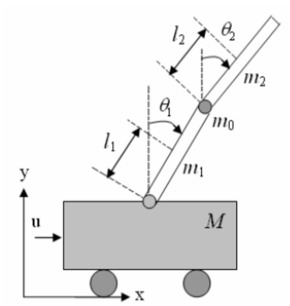


Fig. 1. Physical model of CDPS

In the previous researches (Li and Tu, 2003; Li, et al., 2004), a multi-model controller based on the Human Simulated Intelligence Control (HSIC) is proposed to provide a practical solution. However, their study utilizes the traditional trial-and-error way for the parameterization process. It is very time-consuming especially when the selected parameter set is required to optimally meet multiple various criteria such as the smaller displacement of the cart over the rail and the less swing numbers during operations.

Our work is to adopt the improved work-flow which formulates the multiobjective equations of the controller for various criteria and provides the automated and efficient solution. Furthermore, a new multiobjective genetic algorithm which extends from the original method of inequality-based multiobjective genetic algorithm (MMGA) (Zakian, et al., 2005) is proposed to solve the multiobjective controller design problem and a test case is studied to validate the multi-performance indices according to the specifications.

2. HSIC INTELLIGENT CONTROL BASED ON SENSORY-MOTOR INTELLIGENT SCHEMA

HSIC bases on Sensory-Motor Intelligent Schema (SMIS) which believes that human super control capability is dependent on Sensory-motor Intelligence (SMI) of all levels of central nerve system. By processing perceived information structurally, human-body control system can perceive the status and rules of the environment around him, adjust human behaviour modes, and foresee human influences on the environment. The intelligent module can go into environment, abstract and symbolize the relation between the environment and the self-body in the subjective world, is called as "Sensory-Motor Intelligent Schema (SMIS)".

The mathematic model of HSIC controller can be seen as SMIS. The SMIS, suitable for movement control of a complex system, is expressed as a triple-element group as follows:

$$S_{KG} = (S_P , S_M , S_A) \tag{1}$$

where

$$S_p = (R, Q, K, \otimes, \Phi),$$

$$S_M = (R, P, L, \Psi, U),$$

$$S_A = \Omega_1 : \Phi_1 \to \Psi_1 ,$$

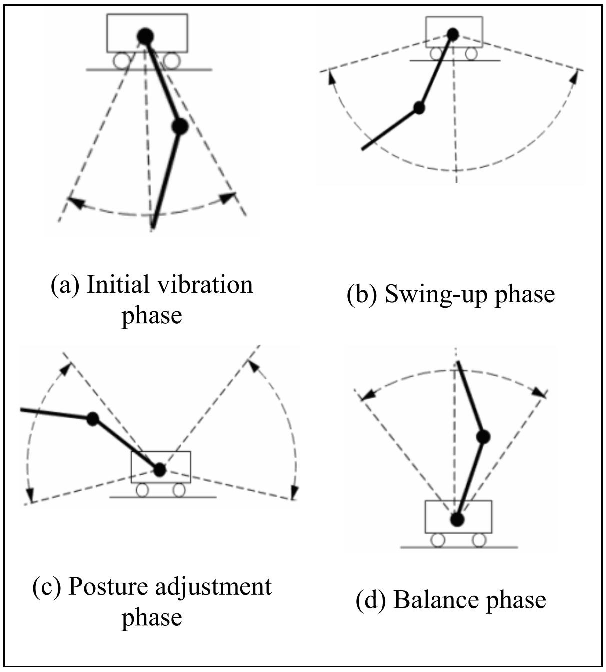
in which $\Omega = \{\omega_1, \omega_2, \cdots, \omega_k\}$,

ω_j : IF φ_j THEN ψ_j .

R is an input variable set; Q is a characteristic element set; K is a relation matrix; $\otimes$ is an operator set; Φ is a characteristic mode set. By means of operations such as "AND" and "OR", the characteristic elements constitute the characteristic mode, which is a higher order SP. The set of all characteristic modes is entitled as a characteristic model: $\Phi=\{\varphi_1,\varphi_2,\cdots,\varphi_r\}$. The relation of the characteristic model Φ and characteristic element set Q is: $\Phi = K \otimes Q$ SM can be described by the following expression with a five-element group: R is an input variable set； $P=[p_1 \quad p_2 \quad \cdots \quad p_n]^T$ is control (decision-making) mode element vector；L is a $r\times n$ order relation matrix (decision-making), and the element l_{ij} in the matrix can be -1, 0, and 1, which respectively represents negative, zero, or positive. Or l_{ij} can also be: $\wedge$, $\vee$, 0, which respectively represents "AND", "OR", or "UNRELATEDNESS". $\Psi=\{\psi_1, \psi_2, \cdots \psi_r\}$ is a control mode set, which indicates quantitative or qualitative relation between control output U and input R. U is a control output variable set. If the relation between the control (decision making) mode element set P and the input set R is quantitative, then the control (decision-making) mode set Ψ can be described by $\Psi : U = L P$ where L and P multiply each other by general matrix multiplication. If the relation between P and R is qualitative, Ψ can be described by a set of productive rules.

3. THE DESIGN OF HSIC CONTROLLER

According to the kinematics characteristics, the whole motion of CDPS can be divided into four phases shown in Fig. 2. The respective control modes for different phases are described as follows and the whole controller diagram is shown in Fig. 3.



(a) Initial vibration phase

(b) Swing-up phase

(c) Posture adjustment phase

(d) Balance phase

Fig. 2. Four kinematics phases of CDPS

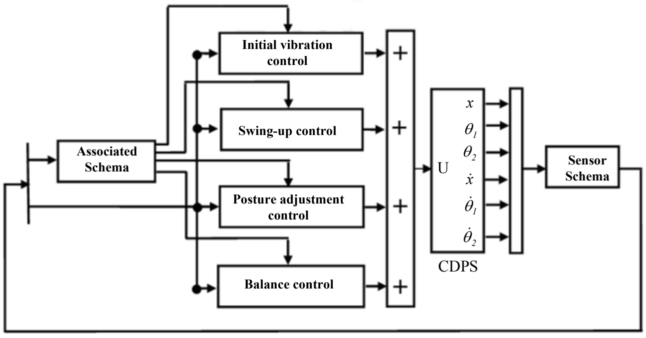


Fig. 3. Structure of a swing-up and handstand controller

3.1 Initial vibration control

Given a minor acceleration to drive the cart, and make the cart move back and forth, the pendulum swings up with the inertia. The initial input U_0 is the minor acceleration. And then energy is pumped into the system at the value $u=k_i\cdot U_0$ (i=1, 2) according to characteristic information about current kinetic status. We consider characteristic modes Eq. (2) and control modes Eq. (3). According to Eq. (1), (2) and (3), we can obtain Eq. (4) which denotes inferential rule set of initial vibration control. The SMIS of initial vibration control can be designed as follows:

$$\Phi_1=\{\phi_{11},\ \phi_{12},\ \phi_{13}\}, \tag{2}$$

where

ϕ_{11} : $t<t_0$ *and* $\theta_1=\theta_2=-\pi$,

ϕ_{12} : $t\ge t_0, |\theta_1|<\theta_{c,1}, |\theta_2|<\theta_{c,1}$ *and* $\dot{\theta}_1\cdot\dot{\theta}_2>0$,

ϕ_{13} : $t\ge t_0, |\theta_1|<\theta_{c,1}, |\theta_2|<\theta_{c,1}$ *and* $\dot{\theta}_1\cdot\dot{\theta}_2<0$,

in which $\theta_{c,1}$ denotes characteristic angle.

$$\Psi_1=\{\psi_{11},\ \psi_{12},\ \psi_{13}\}, \tag{3}$$

where

ψ_{11} : $u=U_0$,
ψ_{12} : $u=k_1U_0$,
ψ_{13} : $u=-k_2U_0$.

$$\Omega_1=\{\omega_{11},\ \omega_{12},\ \omega_{13}\}, \tag{4}$$

where

ω_{11} : $\phi_{11}\rightarrow\psi_{11}$,
ω_{12} : $\phi_{12}\rightarrow\psi_{12}$,
ω_{13} : $\phi_{13}\rightarrow\psi_{13}$.

From Eq. (2) to (4), we can obtain Eq. (5), (6) and (7).

$$\omega_{11}:\ IF \quad t<t_0\ and\ \theta_1=\theta_2=-\pi \quad THEN \quad u=U_0. \tag{5}$$

$$\omega_{12}:\ IF \quad t\ge t_0, |\theta_1|>\theta_{c,1}, |\theta_2|>\theta_{c,1}\ and\ \dot{\theta}_1\cdot\dot{\theta}_2>0 \quad THEN \quad u=k_1\cdot U_0. \tag{6}$$

$$\omega_{13}:\ IF \quad t\ge t_0, |\theta_1|>\theta_{c,1}, |\theta_2|>\theta_{c,1}\ and\ \dot{\theta}_1\cdot\dot{\theta}_2<0 \quad THEN \quad u=-k_2\cdot U_0. \tag{7}$$

The SMIS of initial vibration phase is shown as follows:

$$S_{KG1}=\begin{cases}u=U_0 & t<t_0,\theta_1=\theta_2=-\pi \\ u=k_1\cdot U_0 & t\ge t_0,|\theta_1|>\theta_{c,1},|\theta_2|>\theta_{c,1},\dot{\theta}_1\cdot\dot{\theta}_2>0 \\ u=-k_2\cdot U_0 & t\ge t_0,|\theta_1|>\theta_{c,1},|\theta_2|>\theta_{c,1},\dot{\theta}_1\cdot\dot{\theta}_2<0\end{cases} \quad (8)$$

3.2 Swing-up control

When the angles of two rods are greater than a certain value, enough energy will be injected into the CDPS so as to enter the posture adjustment phase and then the balance control phase in turn. Because the kinetic characteristics of pendulum above and below the horizontal line are opposite, control effects with opposite direction should are applied to the cart. Pulling control mode or pushing mode is switched according to the angle error and the moving trend of two rods. Using a larger input force, pulling or pushing, makes two rods swing up at the same time. Consider characteristic modes Eq. (9) and control modes Eq. (10). According to Eqs. (1), (9) and (10), we can obtain Eq. (11) which denotes the inferential rule set of swing-up control. The SMIS of swing-up control can be designed as follows:

$$\Phi_2=\{\phi_{21},\ \phi_{22}\}, \quad (9)$$

where

$$\phi_{21}\ :\ \theta_1>\theta_{c,2}\ ,$$
$$\phi_{22}\ :\ \theta>\theta_{c,3}(\theta_{c,3}>-\frac{\pi}{2}).$$

in which $\theta_{c,2}$ and $\theta_{c,3}$ denote characteristic angles.

$$\Psi_2=\{\psi_{21},\ \psi_{22}\}, \quad (10)$$

where

$$\psi_{21}\ :\ u=U_1\ ,$$
$$\psi_{22}\ :\ u=-U_2,$$

$$\Omega_2=\{\omega_{21},\ \omega_{22}\}, \quad (11)$$

where

$$\omega_{21}\ :\ \phi_{21}\rightarrow\psi_{21},$$
$$\omega_{22}\ :\ \phi_{22}\rightarrow\psi_{22.}$$

From Eqs. (9) to (11), we can obtain Eqs. (12) and (13).

$$\omega_{31}:\ IF\quad \theta_1>\theta_{c,2}\quad THEN\quad u=U_1. \quad (12)$$

$$\omega_{31}:\ IF\quad \theta_1>\theta_{c,3},(\theta_{c,3}>-\frac{\pi}{2})\quad THEN\quad u=-U_2. \quad (13)$$

The SMIS of swing-up control phase is shown as follows:

$$S_{KG2}=\begin{cases}u=U_1 & \theta_1>\theta_{c,2}, \\ u=-U_2 & \theta_1>\theta_{c,3},(\theta_{c,3}>-\frac{\pi}{2})\end{cases}. \quad (14)$$

3.3 Posture adjustment control

For linearizing the CDPS at two different angle $\theta_{c,4}$ and $\theta_{c,5}$, two sets of LQR (Lewis and Syrmos, 1995) parameters are figured out according to the linear model of the CDPS.

With the set of LQR parameters, the SMIS adjusts relative posture and motion trend between inner and outer rod, so that the posture for the two rods becomes a quasi-line. Consider characteristic modes Eq. (15) and control modes Eq.(16) and according to Eqs. (1), (15) and (16), we can obtain Eq.(17) which denotes the inferential rule set of posture adjustment control. The SMIS of posture adjustment control can be designed as follows:

$$\Phi_3=\{\phi_{31},\ \phi_{32}\}, \quad (15)$$

where $\phi_{31}\ :\ |\theta_1|>\theta_{c,4}$, $\phi_{32}\ :\ |\theta_1|>\theta_{c,5}$

in which $\theta_{c,4}$ and $\theta_{c,5}$ denote characteristic angles.

$$\Psi_3=\{\psi_{31},\ \psi_{32}\}, \quad (16)$$

where $\psi_{31}\ :\ u=K_{LQR,1}\cdot X$, $\psi_{32}\ :\ u=K_{LQR,2}\cdot X$

in which $K_{LQR,1}$ and $K_{LQR,2}$ denotes optimal feedback gain respectively.

$$X=[x\ \ \theta_1\ \ \theta_2\ \ \dot{x}\ \ \dot{\theta}_1\ \ \dot{\theta}_2]^T, \quad (17)$$
$$\Omega_3=\{\omega_{31},\ \omega_{32}\},$$

where $\omega_{31}\ :\ \phi_{31}\rightarrow\psi_{31},\ \omega_{32}\ :\ \phi_{32}\rightarrow\psi_{32}$.

From Eq. (15) to (17), we can obtain Eq. (18) and (19).

$$\omega_{31}:\ IF\quad |\theta_1|>\theta_{c,4}\quad THEN\quad u=K_{LQR,1}\cdot X \quad (18)$$

$$\omega_{32}:\ IF\quad |\theta_1|>\theta_{c,5}\quad THEN\quad u=K_{LQR,2}\cdot X \quad (19)$$

The SMIS of posture adjustment phase is shown as follows:

$$S_{KG3}=\begin{cases}u=K_{LQR,1}\cdot X & |\theta_1|>\theta_{c,4}, \\ u=K_{LQR,2}\cdot X & |\theta_1|>\theta_{c,5}.\end{cases} \quad (20)$$

3.4 Balance control

In this control process, LQR control is applied to make cart return to initial position and keep two rods steadily at the upward equilibrium position. Around equilibrium position, assume $\sin\theta_1\approx\theta_1$, $\sin\theta_2\approx\theta_2$, $\cos\theta_1\approx 1$, $\cos\theta_2\approx 1$, $\theta_1\approx 0$ and $\theta_2\approx 0$. Consider characteristic mode Eq. (21) and control mode Eq. (22), according to Eq. (1), (21) and (22), we obtain Eq. (23) which denotes the inferential rule set of balance phase. The SMIS of LQR controller is designed as follows:

$$\Phi_4=\{\phi_{41}\}, \quad (21)$$

where $\phi_{41}:|\theta_1|>\theta_{c,6}\ and\ |\theta_2|>\theta_{c,6}$,in which $\theta_{c,6}$ denotes characteristic angle.

$$\Psi_4=\{\psi_{41}\}, \quad (22)$$

where $\psi_4:u=K_{LQR,3}\cdot X$, in which $K_{LQR,1}$ denotes optimal feedback gain.

$$\Omega_4 = \{\omega_{41}\} \tag{23}$$

where ω_{41} : $\phi_{41} \rightarrow \psi_{41}$.

From Eqs. (21) to (23), we can obtain Eq. (24).

$$\begin{aligned} \omega_{41}:\ & IF \quad |\theta_1| > \theta_{c,6} \ and \ |\theta_2| > \theta_{c,6} \\ & THEN \quad u = K_{LQR,3} \cdot X \end{aligned} \tag{24}$$

The SMIS of Balance control phase is shown as follows:

$$S_{KG4} = \{ \ u = K_{LQR,3} \cdot X \qquad |\theta_1| > \theta_{c,6}, |\theta_2| > \theta_{c,6} \tag{25}$$

Combining S_{KG1}, S_{KG2}, S_{KG3} and S_{KG4} of four SMIS, we have the following general SMIS of SUHC of a CDPS:

$$S_{KG} = \begin{cases} \begin{cases} u = U_0 & t < t_0, \theta_1 = \theta_2 = -\pi \\ u = k_1 \cdot U_0 & t \geq t_0, |\theta_1| > \theta_{c,1}, |\theta_2| > \theta_{c,1}, \dot{\theta}_1 \cdot \dot{\theta}_2 > 0 \\ u = -k_2 \cdot U_0 & t \geq t_0, |\theta_1| > \theta_{c,1}, |\theta_2| > \theta_{c,1}, \dot{\theta}_1 \cdot \dot{\theta}_2 < 0 \end{cases} \\ \begin{cases} u = U_1 & \theta_1 > \theta_{c,2} \\ u = -U_2 & \theta_1 > \theta_{c,3}, (\theta_{c,3} > -\frac{\pi}{2}) \end{cases} \\ \begin{cases} u = K_{LQR,1} \cdot X & |\theta_1| > \theta_{c,4} \\ u = K_{LQR,2} \cdot X & |\theta_1| > \theta_{c,5} \end{cases} \\ \{ u = K_{LQR,3} \cdot X \quad |\theta_1| > \theta_{c,6}, |\theta_2| > \theta_{c,6} \end{cases} \tag{26}$$

where u is limited by $u_{\max}$ and satisfies the following condition

$$\text{If } |u| \geq u_{\max} \text{ then } u = \text{sgn}(u) \times u_{\max} \tag{27}$$

In Eq. (27), the expressions on the left side are control modes of each sub-controller; the ones on the right side are the corresponding characteristic states.

4. MMGA EXTENSION FOR THE EFFICIENTLY AUTOMATED SOLUTION

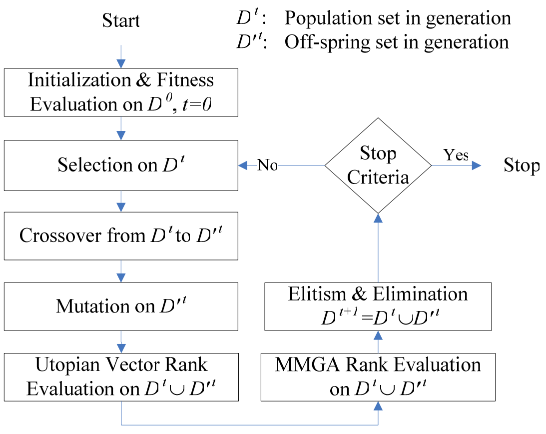


Fig.4. The flow chart of MMGA Extension

Genetic algorithms, first introduced by Holland, were later improved by many researchers (Leung and Wang, 2001; Tsai, et al., 2004). Due to the global explorer capabilities, MMGA utilizes GA as the solution explorer and provides a multiobjective reasoning approach to search the optimal parameterizations for MOI problems (Chou, et al., 2008). The original MMGA proposes an auxiliary vector performance index which would be related to the MOI set of the controller design specifications and employs the global search capability of genetic algorithms according to the heuristic Pareto-ranking rules.

An extension for the convergence characteristics is performed by inviting a utopian vector which is used to guide the centralized search direction and can contribute the better convergence performance. The extension scheme is shown in Fig. 4.

4.1 Population Initialization

The hybrid coding representation of integer and float-point is used here and each chromosome is encoded as a vector of floating-point and integer numbers, with the same length. The hybrid coding representation is accurate and efficient because it is closest to the practical design space. Initially, 16 parameters ordered as {a1, …, a5, b1, …, b6, c1, …, c3} are needed and they would be encoded as the float-point genes of the chromosome.

The meaning of each gene is described as follows:

(1) a1, a2, …, a5: represent five control coefficients in float-point coding representation of U_0, k_1, k_2, U_1 and U_2, respectively in Eq. (26).

(2) b1, b2, …, b6 : represent four characteristic schemas in real coding representation of $\theta_{c,1}$, $\theta_{c,2}$, $\theta_{c,3}$, $\theta_{c,4}$, $\theta_{c,5}$ and $\theta_{c,6}$, respectively in Eq.(26).

(3) c1, c2,c3 : represent three LQR parameters $K_{LQR,1}$, $K_{LQR,2}$ and $K_{LQR,3}$, and use orthogonal arrays planning parameter solution combinations (Montgomery, 1991) in integer coding representation, respectively in Eq. (26).

The initialization procedure produces M chromosomes, where M denotes the population size, in the following steps.

Step 1: Generate a random value $\beta \in [0,\ 1]$.

Step 2: Let, $x_i = l_i + \beta(u_i - l_i)$, where l_i and u_i are the domain of x_i . Repeat N times and produce a vector $(x_1, x_2, ..., x_N)$.

Repeat the steps by M times and produce M initial feasible solutions.

4.2 Fitness Assignment

The evolutionary population in the proposed algorithm would be divided into two parts: the former part would hold a competition tournament according to the distance from the utopian vector and the latter part follows the original MMGA approach. Such the balance design speeds up the search performance and reserve the Pareto variety simultaneously.

The detailed extension rule is to design a customized separation ratio $v \in [0,1]$. Suppose that the estimated population size is M, the rank strategy would evaluate the first $M \times v$ individuals with the distance from the utopian vector and the losers of size $M \times (1-v)$ are directly to follow MMGA rank rules. If $v = 1.0$, the extended algorithm directly evolutes toward unique utopian solution and in opposition, if $v = 0.0$, the extended version would operate just same to the original MMGA. And the final rank result is set as the fitness value.

4.3 Crossover Operation

The crossover operator used here is the uniform crossover (Gen and Cheng, 1997) integrated with an arithmetical operator derived from convex set theory, which randomly select multi-points, exchange the selected parts of two parents, randomly select multi-points and calculate the linear combinations at these selected genes to generate new offspring. For example, let two parents be $x = (x_1, x_2, \dots, x_N)$ and $y = (y_1, y_2, \dots, y_N)$. If they are crossed after the randomly selected positions besides calculating the linear combinations at k_{th} position selected among selected positions after previously step, the resulting offspring are

$$x' = (x_1,\ y_2,\ \dots,\ y'_k,\ x_{k+1},\ y_{k+2},\ \dots,\ y_N),$$

$$y' = (y_1,\ x_2,\ \dots,\ x'_k,\ y_{k+1},\ x_{k+2},\ \dots,\ x_N),$$

where

$x'_k = x_k + \beta(y_k - x_k)$ $y'_k = l_k + \beta(u_k - l_k)$, l_k and u_k are the domain of y_k, and β is random value, in which $\beta \in \{0,\ 0.1,\ 0.2, \dots, or\ 1\}$. If x' and y' are in the integer coding, choose the integer values which are closer to them (Tsai, et al., 2004).

4.4 Mutation Operation

Mutation operation used here is multi-point mutation integrated with an arithmetical operator derived from convex set theory. Multi-genes in a single chromosome are randomly chosen to execute the mutation of convex combination. For a given $x = (x_1, x_2, \dots, x_i, x_j, x_k, \dots, x_N)$, if the elements x'_i and x'_k are randomly selected for mutation, the resulting offspring is $x' = (x_1, x_2, \dots, x'_i, x_j, x'_k, \dots, x_N)$. The two new genes and are $x'_i = l_i + \beta(u_i - l_i)$, $x'_k = l_k + \beta(u_k - l_k)$, l_i and u_i are he domain of x'_i, l_k and u_k are he domain of x'_k and β is random value, in which $\beta \in \{0,\ 0.1,\ 0.2, \dots,\ or\ 1\}$. If x'_i and x'_k are in the integer coding, choose the integer values which are closer to them.

4.5 Diversity Operation

The diversity maintenance in MMGA is proposed by (Liu, 1997) to solve multiobjective problems. The diversity operation is performed after the crossover and mutation operation. And the hamming distance evaluation is employed here to watch out the part-wise closeness for the premature avoidance. First step is to sort the population containing parents and children. The population can be divided into two subpopulations called preserved population and eliminated population.

For a given chromosome $X_i = (x_1, x_2, \dots, x_k, \dots, x_N)$, if it belongs to the preserved population, we calculate its hamming distance with other preserved chromosomes according Eq.(34).

$$d_i = \|X_i - X_1\| = \sqrt{\sum_{k=1}^{N} (x_{ik} - x_{1k})^2} \quad (i = 2, 3, \dots, M) \qquad (26)$$

where x_{ik} is the *k*th gene of the *i*th chromosome. Setting a threshold distance L. If d_i < L, the *i*th chromosome would be replaced by arbitrary one in eliminated population. A probability variable p_d is used to decide the execution of the diversity operation.

5. MULTIOBJECTIVE CASE STUDY

The physical parameters of CDPS for the study case are shown in Table 1.

Table 1. Physical parameters of CDPS

Mass of cart (M)	1.32kg
Mass of encoders (m0)	0.208kg
Mass of two pendulums (m1 ,m2)	0.108 kg
Half length of two pendulums (l1,l2)	0.2m
Length of two pendulums (L)	0.4m
Displacement of cart (x)	(-0.3,0.3) m
friction coefficient of cart(f0)	22.915 N.s/m
friction coefficient of pendulums (f1,f2)	0.7756 N.s.m
Moment of inertia of pendulums (J1,J2)	0.00144 kg.m2

The various objectives and their relative parameter definitions are derived from the test problems in (Whidborne, et al., 1994). For verifying the proposed method, the authors design another HSIC controller by rewriting Eq. (26) to Eq. (28) and compare the behaviour of the two HSIC controllers.

$$S_{KG} = \begin{cases} \begin{cases} u = U_0 & t < t_0, \theta_1 = \theta_2 = -\pi \\ u = k_1 \cdot U_0 & t \ge t_0, |\theta_1| > \theta_{c,1}, |\theta_2| > \theta_{c,1}, \dot{\theta}_1 \cdot \dot{\theta}_2 > 0 \\ u = -k_2 \cdot U_0 & t \ge t_0, |\theta_1| > \theta_{c,1}, |\theta_2| > \theta_{c,1}, \dot{\theta}_1 \cdot \dot{\theta}_2 < 0 \end{cases} \\ \begin{cases} u = U_1 & \theta_1 > \theta_{c,2} \\ u = -U_2 & \theta_1 > \theta_{c,3}, (\theta_{c,3} > -\frac{\pi}{2}) \end{cases} \\ \{ u = K_{LQR,1} \cdot X \quad |\theta_1| > \theta_{c,4} \\ \{ u = K_{LQR,3} \cdot X \quad |\theta_1| > \theta_{c,5}, |\theta_2| > \theta_{c,5} \end{cases} \qquad (28)$$

The performance indices of the CDPS are described as follows:

(1) ϕ_1 is sum of displacement of a cart over the rail boundary. And it is formulated as

$$\phi_1 = \min\left\{\sum_{i=1}^{m} \delta_i(t)\right\}, \qquad (29)$$

$$\delta_i(t) \equiv \begin{cases} 0 & (\text{if } |X_i(t)| \le \varphi) \\ |X_i(t)| - \varphi & (\text{if } |X_i(t)| > \varphi) \end{cases}$$

where φ is the boundary, m is a positive integer specified by the designer.

(2) ϕ_2 is the number of swings, and can be formulated as

ϕ_2 = k, where k is a positive integer. (30)

(3) ϕ_3 is the settling time ($\pm 2\%$), and can be formulated as

$$\phi_3 = \min\left\{ t \text{ such that } \frac{|\hat{x}(h) - x(\infty, h)|}{|x(\infty, h)|} \le 2\% \right\}, \qquad (31)$$

where $\hat{x}(h) = \sup_{t \geq 0} |x(t,h)|$.

(4) ϕ_4 is the settling time ($\pm 2\%$), and can be formulated as

$$\phi_4 = \min\left\{ t \ \text{ such that } \ \frac{|\hat{\theta}_1(h) - \theta_1(\infty, h)|}{|\theta_1(\infty, h)|} \leq 2\% \right\}, \qquad (32)$$

where $\hat{\theta}_1(h) = \sup_{t \geq 0} |\theta_1(t,h)|$.

The cart double pendulum system specifications are set according to a number of performance requirements: $\phi_1 \leq 7$, $\phi_2 \leq 12$, $\phi_3 \leq 13$, and $\phi_4 \leq 13$. And the evolution parameters are used in the algorithm containing: the population size is 100, the numbers of generation is 1000, the crossover rate is 0.9, the mutation rate is 0.05, the diversity rate is 0.1 and hamming distance threshold is set as 0.5. The simulation experimental result is show in Table 2 and the response of CDPS and control effect of SMIS is show in Fig. 5 and 6.

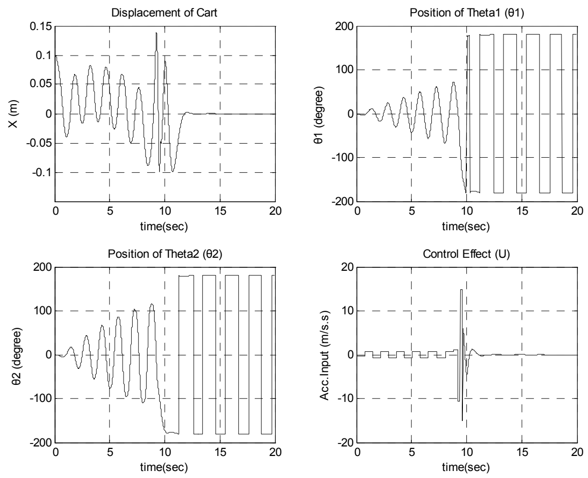


Fig. 5. Response and control effect in φ= 0.1

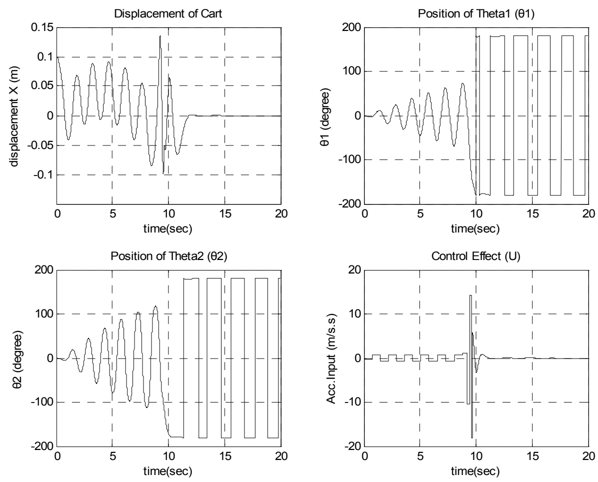


Fig. 6. Response and control effect in φ= 0.1

Table 2. Summary of Simulation Results

SMIS	ϕ_1	ϕ_2	ϕ_3	ϕ_4	FIGURE
Eq.(26)	4.401	12	12.325	10.830	Fig. 5
Eq.(28)	4.142	12	12.440	10.040	Fig. 6

6. CONCLUSION

In this article, a new multiobjective optimization genetic algorithm is successfully applied to solve the CDPS control problem. To illustrate the effectiveness of the proposed method, a multiobjective case study is designed to validate the solution. The experiment results show that the responses of CDPS can achieve the multi-performance indices according to the proposed method.

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