Constraint handling for State Dependent Parameter models

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Abstract: This paper considers Proportional–Integral–Plus (PIP) control of nonlinear dynamic systems described by State Dependent Parameter (SDP) models with constraints. More specifically, a low level stabilising SDP/PIP controller is first developed to steer the system in the desired direction, whilst a Reference Governor (RG) is subsequently introduced to account for constraints in the system variables. This contrasts with the (off-line) simulation-based methods previously used for PIP control of SDP models with constraints. Furthermore, the particular parametrisation of the RG used in this paper provides useful insight into, and quantification of, the effects of the constraints on the nonlinear control system.

Keywords: state dependent parameters; constraints; reference governor

1. INTRODUCTION

The State Dependent Parameter (SDP) methodology (see Young et al., 2001, and references therein) is based on the definition of quasi–linear state space models whose parameters depend on elements of the state vector. Within the framework of Non–Minimal State Space (NMSS) design (e.g. Young et al., 1987; Taylor et al., 2000), this effectively means that the system parameters are functions of previous measurements of the input or output variables. Here, there are close parallels with Time Variable Parameter (TVP) models. However, the parameters of TVP systems are assumed to vary slowly, while the state dependent parametrisation of SDP models allows for rapid changes in the system dynamics. Hence, SDP models describe non–linear processes that can include chaotic systems and systems that have previously been modelled using a bilinear approach (Dunoyer et al., 1997).

A variety of methods have been presented for the identification and estimation of SDP models (Åkesson and Toivonen, 2006; Toivonen, 2003; Toivonen et al., 2007; Young et al., 2001; Taylor et al., 2007a). Furthermore, the linear–like structure of SDP models allows for them to be considered at each sampling instant as *frozen*, linear instances of the non–linear system. This formulation is used to define a NMSS model and hence design a Proportional–Integral–Plus (PIP) control law at each sampling instant, using linear methods such as pole assignment (Young, 1996; McCabe et al., 2000; Taylor et al., 2008). Recent publications have demonstrated advantages of the approach for practical applications (Stables and Taylor, 2006; Taylor et al., 2006, 2007b). However, in all the above cases, any system constraints were dealt with using an off–line (simulation based) trial and error approach.

By contrast, the present paper evaluates the predictive reference management technique of the Reference Governor (RG) (extensively studied in Bemporad et al. (1997) for linear and Bemporad (1998) for non-linear systems) to allow for system constraints to be handled inherently within the design of the controller. In particular, it extends the SDP/PIP control design methodology to exploit RG methods and hence allow for on-line constraint handling. The parametrisation of the degrees–of–freedom presented here is a combination of the RG and Closed Loop Paradigm (CLP) (Rossiter, 2003), and hence provides a clear insight into, and quantification of, the effect of the constraints on the closed loop dynamics.

The following Section 2 introduces SDP/PIP control, while Section 3 develops the proposed RG approach for constraint handling. Section 4 demonstrates the application of the RG with a simulation example, with the conclusions presented in Section 5.

2. BACKGROUND

The description of the general SDP system can be found in Young et al. (2001). For control system design, the following subset of the general model is particularly useful, since it limits the dependency of the system parameters to be a direct function of the input and output signals. Such a SDP system can be described by the following difference equation, in which the parameters change at each sampling instant:

$$y_k = -\sum_{i=1}^{n} a_i(\chi_{i,k}) y_{k-i} + \sum_{i=1}^{m} b_i(\psi_{i,k}) u_{k-i} \qquad (1)$$

where y_k and u_k are the system output and input at sample k respectively, while $\chi_{i,k}$ and $\psi_{i,k}$ are typically functions of their lagged values. For simplicity of notation, in the following $a_i(\chi_{i,k})$ and $b_i(\psi_{i,k})$ are referred to as $a_{i,k}$ and $b_{i,k}$ respectively.

The SDP model above can be described by the following piecewise linear state space form:

$$\boldsymbol{x}_{k+1} = \mathbf{A}_k\boldsymbol{x}_k + \boldsymbol{b}_k u_k + \boldsymbol{d} r_{k+1} \tag{2a}$$
$$y_k = \boldsymbol{c}\boldsymbol{x}_k \tag{2b}$$

where r_k is the reference input. The state vector is subsequently defined as:

$$\boldsymbol{x}_k = [y_k \quad y_{k-1} \quad \cdots \quad y_{k-n+1} \quad u_{k-1} \quad u_{k-2} \quad \cdots \quad u_{k-m+1} \quad z_k]$$

were $z_k = \frac{1}{1-z^{-1}}(r_k - y_k)$ is the integral-of-error state variable introduced for steady state tracking (see for example Taylor et al. (2000) or Exadaktylos et al. (2006) for a demonstration of the steady state tracking property due to the integral-of-error state variable).

Finally, the system matrices are defined as:

$$\mathbf{A}_k = \begin{bmatrix} -a_{1,k} & \cdots & -a_{n,k} & b_{2,k} & \cdots & b_{m,k} & 0 \\ 1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ a_{1,k} & \cdots & a_{n,k} & -b_{2,k} & \cdots & -b_{m,k} & 1 \end{bmatrix}$$

$$\boldsymbol{b}_k = [b_{1,k} \ 0 \ \cdots \ 0 \ 1 \ 0 \ \cdots \ 0 \ -b_{1,k}]^T$$
$$\boldsymbol{d} = [0 \ 0 \ \cdots \ 0 \ 0 \ 0 \ \cdots \ 0 \ 1]^T$$
$$\boldsymbol{c} = [1 \ \cdots 0 \ 0 \ 0 \ \cdots \ 0 \ 0 \ 0]$$

Various approaches to SDP/PIP control have been suggested using an analogy with linear methods. With regard to (suboptimal) linear quadratic design, earlier research has typically solved the discrete-time algebraic Riccati equation at each sampling instant (Taylor et al., 2007b), i.e. a state dependent Riccati equation approach (Banks et al., 2007). For pole assignment, Stables and Taylor (2006) derive algebraic solutions for the simplest case of a single sample time delay (i.e. $\tau = 1$), whilst Taylor et al. (2008) generalise these results for the $\tau \geq 1$ case. The latter approach reduces the computational burden in practical applications. For all these methods, the state feedback controller takes the form:

$$u_k = -\boldsymbol{k}_k\boldsymbol{x}_k \tag{3}$$

where the state gain vector $\boldsymbol{k}_k$ is varying to account for the system non–linearity. A full description is omitted here for brevity – see above citations for details.

For pole assignment design after application of the above control law, it can be shown (Taylor et al., 2008) that the system equations (2) take the form:

$$\boldsymbol{x}_{k+1} = \mathbf{A}\boldsymbol{x}_k + \boldsymbol{d} r_{k+1} \tag{4a}$$
$$y_k = \boldsymbol{c}\boldsymbol{x}_k \tag{4b}$$

with constant system matrices. More specifically, since the $\boldsymbol{k}_k$ vector changes at each sampling instant to achieve the closed loop performance objectives, the $\mathbf{A} = \mathbf{A}_k - \boldsymbol{b}_k\boldsymbol{k}_k$ matrix is now constant and defined by the design objectives.

Finally, to avoid integral windup problems, the controller is usually implemented in incremental form (see Taylor et al., 2004), as shown below:

$$\boldsymbol{u}_k = \boldsymbol{u}_{k-1} - \boldsymbol{k}_k\Delta\boldsymbol{x}_k \tag{5a}$$

with the following corrections:

$$\boldsymbol{u}_k = \begin{cases} \boldsymbol{u}_{k-1} + \overline{\boldsymbol{\Delta u}} & , -\boldsymbol{k}_k\Delta\boldsymbol{x}_k > \overline{\boldsymbol{\Delta u}} \\ \boldsymbol{u}_{k-1} - \underline{\boldsymbol{\Delta u}} & , -\boldsymbol{k}_k\Delta\boldsymbol{x}_k < \underline{\boldsymbol{\Delta u}} \end{cases} \tag{5b}$$

$$\boldsymbol{u}_k = \begin{cases} \overline{\boldsymbol{u}} & , \boldsymbol{u}_k > \overline{\boldsymbol{u}} \\ \underline{\boldsymbol{u}} & , \boldsymbol{u}_k < \underline{\boldsymbol{u}} \end{cases} \tag{5c}$$

where $\Delta\boldsymbol{x}_k = \boldsymbol{x}_k - \boldsymbol{x}_{k-1}$.

3. THE REFERENCE GOVERNOR APPROACH

The underlying idea of the RG is to predict the system evolution, detect any constraint violations and generate a 'slack' reference sequence that yields constraint satisfaction in the closed-loop response. This slack reference should converge to the externally specified reference (or as close as possible depending on the system constraints) so that there is steady state tracking of the desired level. Although there are numerous options on how to parametrise this slack reference sequence (e.g. Bemporad et al., 1997; Bemporad, 1998; Oh and Agrawal, 2005), the present paper utilises a perturbation around the actual reference as follows:

$$w_k = r_k + \beta_k \tag{6}$$

where w_k is the artificial reference applied to the system. The above parametrisation closely resembles the parametrisation used by the Closed Loop Paradigm (Rossiter, 2003), where the degrees–of–freedom are parametrised as perturbations around the control input signal. As illustrated later for the simulation example, the above parametrisation of w_k can also provide a measure of the effect of the constraints on the closed loop system.

Using the parametrisation (6), the aim is to minimise the perturbations around the reference, while maintaining the system within the region defined by the constraints. In this regard, the following constrained optimisation problem is solved at each sampling instant:

$$\min_{\beta_1,\ldots,\beta_{N_p}} \quad \sum_{i=1}^{N_p} \beta_i^2 \tag{7a}$$

$$\text{subject to:} \begin{cases} \underline{u} \leq u_{k+i} \leq \overline{u} \\ \underline{\Delta u} \leq \Delta u_{k+i} \leq \overline{\Delta u} \\ \underline{y} \leq y_{k+i+1} \leq \overline{y} \end{cases} \tag{7b}$$

for $i = 0, \ldots, N_p - 1$, where N_p is the prediction horizon; u_k is given by (3); $\Delta u_{k+i} = u_{k+i} - u_{k+i-1}$ is the control increment at each sampling instant; the system is described by equation (4), albeit with the reference vector (6) instead of r_k; $\underline{\cdot}$ and $\bar{\cdot}$ refer to minimum and maximum allowed values for the system variables, where the inequalities in the constraints are element by element inequalities. In the absence of constraints, the minimising β-sequence is zero and the reference remains unchanged resulting in the desired closed–loop dynamics. However, when the actual reference signal r_k would result in constraint violation by the control system within the prediction horizon, the RG intervenes and adjusts w_k so that the constraints are satisfied. In this manner, the amplitude of β also provides a measure of the effect of the constraints on the closed loop control system.

For commercial optimisation tools to be used, the above constraint optimisation problem is cast in a more compact matrix form. In this regard, the optimisation parameters are formed into a single vector as:

$$\boldsymbol{\beta} = \left[\beta_1 \ \beta_2 \ \dots \ \beta_{N_p}\right]^T$$

and the following predicted state, future control input, future control input increment, future reference and predicted output vectors are defined:

$$\begin{aligned}
\boldsymbol{X} &= \left[\boldsymbol{x}_{k+1}^T \ \boldsymbol{x}_{k+2}^T \ \dots \ \boldsymbol{x}_{k+N_p}^T\right]^T \\
\boldsymbol{U} &= \left[u_k \ u_{k+1} \ \dots \ u_{k+N_p-1}\right]^T \\
\boldsymbol{\Delta U} &= \left[\Delta u_k \ \Delta u_{k+1} \ \dots \ \Delta u_{k+N_p-1}\right]^T \\
\boldsymbol{S} &= \left[r_{k+1} \ r_{k+2} \ \dots \ r_{k+N_p}\right]^T \\
\boldsymbol{Y} &= \left[y_{k+1} \ y_{k+2} \ \dots \ y_{k+N_p}\right]^T
\end{aligned}$$

Substituting (6) into (4), the state predictions for the system take the following form:

$$\begin{aligned}
\boldsymbol{x}_{k+1} =& \mathbf{A}\boldsymbol{x}_k + \boldsymbol{d}r_{k+1} + \boldsymbol{d}\beta_1 \\
\boldsymbol{x}_{k+2} =& \mathbf{A}\boldsymbol{x}_{k+1} + \boldsymbol{d}r_{k+2} + \boldsymbol{d}\beta_2 \\
=& \mathbf{A}^2\boldsymbol{x}_k + (\mathbf{A}\boldsymbol{d}r_{k+1} + \boldsymbol{d}r_{k+2}) + \\
& + (\mathbf{A}\boldsymbol{d}\beta_1 + \boldsymbol{d}\beta_2) \\
& \vdots \\
\boldsymbol{x}_{k+N_p} =& \mathbf{A}^{N_p}\boldsymbol{x}_k + \sum_{i=0}^{N_p-1} \mathbf{A}^i \boldsymbol{d}r_{k+N_p-i} + \\
& + \sum_{i=0}^{N_p-1} \mathbf{A}^i \boldsymbol{d}\beta_{k+N_p-i}
\end{aligned}$$

or in a more compact matrix form:

$$\boldsymbol{X} = \mathbf{F}\boldsymbol{x}_k + \mathbf{H}_r\boldsymbol{S} + \mathbf{H}_r\boldsymbol{\beta}$$

in which

$$\mathbf{F} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^{N_p} \end{bmatrix}; \mathbf{H}_r = \begin{bmatrix} \boldsymbol{d} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{A}\boldsymbol{d} & \boldsymbol{d} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}^{N_p-1}\boldsymbol{d} & \mathbf{A}^{N_p-2}\boldsymbol{d} & \cdots & \boldsymbol{d} \end{bmatrix}$$

where $\mathbf{0}$ is a $(n+m)\times 1$ vector of zeros. Next, the following matrices can be defined in order for the future control input vector to be written in a similar matrix form,

$$\mathbf{K}_{1,k} = \begin{bmatrix} -\boldsymbol{k}_k \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \vdots \\ \mathbf{0}^T \end{bmatrix}$$

$$\mathbf{K}_{2,k} = \begin{bmatrix} \mathbf{0}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T & \mathbf{0}^T \\ -\boldsymbol{k}_k & \mathbf{0}^T & \cdots & \mathbf{0}^T & \mathbf{0}^T \\ \mathbf{0}^T & -\boldsymbol{k}_k & \cdots & \mathbf{0}^T & \mathbf{0}^T \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0}^T & \mathbf{0}^T & \cdots & -\boldsymbol{k}_k & \mathbf{0}^T \end{bmatrix}$$

where $\mathbf{k}_k$ is the control gain matrix defined in (3). Note that the gain vector is assumed to be constant throughout the prediction horizon. This follows directly from the 'frozen system' assumption mentioned above and means that the RG methodology can be applied to SDP/PIP systems determined by suboptimal LQ methods (where equation (4) does not necessarily hold), in addition to pole assignment. In either case, at the following sampling instant $k+1$, a new calculation of the control gain vector is determined using the latest values of the system matrices.

The future control input vector can subsequently be written as:

$$\begin{aligned}
\boldsymbol{U} &= \mathbf{K}_{1,k}\boldsymbol{x}_k + \mathbf{K}_{2,k}\boldsymbol{X} \\
&= (\mathbf{K}_{1,k} + \mathbf{K}_{2,k}\mathbf{F})\boldsymbol{x}_k + \mathbf{K}_{2,k}\mathbf{H}_r\boldsymbol{S} + \mathbf{K}_{2,k}\mathbf{H}_r\boldsymbol{\beta}
\end{aligned}$$

In the same manner, and noting that $\boldsymbol{\Delta U} = -\mathbf{C}_1 u_{k-1} + \mathbf{C}_2\boldsymbol{U}$, with:

$$\boldsymbol{c}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}; \mathbf{C}_2 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & -1 & 1 & 0 \\ 0 & 0 & \cdots & 0 & -1 & 1 \end{bmatrix}$$

the future control increment vector is written as:

$$\begin{aligned}
\boldsymbol{\Delta U} = & \boldsymbol{c}_1 u_{k-1} + \mathbf{C}_2 (\mathbf{K}_{1,k} + \mathbf{K}_{2,k}\mathbf{F})\boldsymbol{x}_k \\
& + \mathbf{C}_2\mathbf{K}_{2,k}\mathbf{H}_r\boldsymbol{S} + \mathbf{C}_2\mathbf{K}_{2,k}\mathbf{H}_r\boldsymbol{\beta}
\end{aligned}$$

Finally, defining the $N_p \times (n+m)N_p$ block diagonal matrix $\bar{\mathbf{C}}$, with $\boldsymbol{c}$ on its diagonal, the predicted output vector is written:

$$\begin{aligned}
\boldsymbol{Y} &= \bar{\mathbf{C}}\boldsymbol{X} \\
&= \bar{\mathbf{C}}\mathbf{F}\boldsymbol{x}_k + \bar{\mathbf{C}}\mathbf{H}_r\boldsymbol{S} + \bar{\mathbf{C}}\mathbf{H}_r\boldsymbol{\beta}
\end{aligned}$$

At this point, it is useful to note that the double inequalities of (7b) can be written as two single inequalities in matrix form. To illustrate using the first one:

$$-\mathbf{U} \le -\underline{\mathbf{U}} \tag{8a}$$

$$\mathbf{U} \le \overline{\mathbf{U}} \tag{8b}$$

where $\underline{\mathbf{U}}$ and $\overline{\mathbf{U}}$ are vectors with the minimum and maximum allowed values for the control input u_k. The second and third double inequalities of (7b) can be described in a similar matrix form with $\underline{\boldsymbol{\Delta}\mathbf{U}}$, $\overline{\boldsymbol{\Delta}\mathbf{U}}$, $\underline{\mathbf{Y}}$ and $\overline{\mathbf{Y}}$ appropriately defined. Combining the above, the control problem (7) takes the following matrix form:

$$\min_{\boldsymbol{\beta}} \quad \boldsymbol{\beta}^T\boldsymbol{\beta}$$

$$\text{subject to:} \quad \mathbf{M}\boldsymbol{\beta} \le \boldsymbol{N}$$

where,

$$\mathbf{M} = \begin{bmatrix} -\mathbf{K}_{2,k}\mathbf{H}_r \\ \mathbf{K}_{2,k}\mathbf{H}_r \\ -\mathbf{C}_2\mathbf{K}_{2,k}\mathbf{H}_r \\ \mathbf{C}_2\mathbf{K}_{2,k}\mathbf{H}_r \\ -\bar{\mathbf{C}}r_k\mathbf{H}_r \\ \bar{\mathbf{C}}r_k\mathbf{H}_r \end{bmatrix}$$

$$\boldsymbol{N} = \begin{bmatrix} -\underline{\mathbf{U}} + \left(\mathbf{K}_{1,k} + \mathbf{K}_{2,k}\mathbf{F}\right)\boldsymbol{x}_k + \mathbf{K}_{2,k}\mathbf{H}_r\boldsymbol{S} \\ \overline{\mathbf{U}} - \left(\mathbf{K}_{1,k} + \mathbf{K}_{2,k}\mathbf{F}\right)\boldsymbol{x}_k - \mathbf{K}_{2,k}\mathbf{H}_r\boldsymbol{S} \\ -\underline{\boldsymbol{\Delta}\mathbf{U}} - \boldsymbol{c}_1 u_{k-1} + \mathbf{C}_2\left(\mathbf{K}_{1,k} + \mathbf{K}_{2,k}\mathbf{F}\right)\boldsymbol{x}_k + \mathbf{C}_2\mathbf{K}_{2,k}\mathbf{H}_r\boldsymbol{S} \\ \overline{\boldsymbol{\Delta}\mathbf{U}} + \boldsymbol{c}_1 u_{k-1} - \mathbf{C}_2\left(\mathbf{K}_{1,k} + \mathbf{K}_{2,k}\mathbf{F}\right)\boldsymbol{x}_k - \mathbf{C}_2\mathbf{K}_{2,k}\mathbf{H}_r\boldsymbol{S} \\ -\underline{\mathbf{Y}} + \bar{\mathbf{C}}\mathbf{F}\boldsymbol{x}_k + \bar{\mathbf{C}}\mathbf{H}_r\boldsymbol{S} \\ \overline{\mathbf{Y}} - \bar{\mathbf{C}}\mathbf{F}\boldsymbol{x}_k - \bar{\mathbf{C}}\mathbf{H}_r\boldsymbol{S} \end{bmatrix}$$

and the inequality sign again refers to element by element inequalities.

In this form, the optimisation problem can be solved using commercial software. In particular, the quadprog function of the MATLAB® optimisation toolbox is utilised for the following simulations. Finally, the ubiquitous Receding Horizon Control approach is used. A measurement is performed, the optimisation matrices are updated and the optimisation problem is solved. For the next sampling instant the procedure is repeated.

4. SIMULATION EXAMPLE

Consider a bilinear system described by the following difference equation based on (1):

$$y_k = 0.7 y_{k-1} + y_{k-2} u_{k-1} \tag{9}$$

with $a_{1,k} = a_1 = -0.7$ and $b_{1,k} = y_{k-2}$. The non-minimal state vector is:

$$\boldsymbol{x}_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix}$$

and the system (9) is described by the NMSS form (2) with,

$$\mathbf{A} = \begin{bmatrix} 0.7 & 0 \\ -0.7 & 1 \end{bmatrix} \qquad \boldsymbol{b}_k = \begin{bmatrix} y_{k-1} \\ -y_{k-1} \end{bmatrix}$$

$$\boldsymbol{d} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \boldsymbol{c} = [1 \ 0]$$

As discussed in Section 3 the SDP/PIP control action has the state feedback form (3). In this example, the generic nonlinear pole placement approach of Taylor et al. (2008) is utilised; for this model structure, see also Kontoroupis et al. (2003). Here, the design objective is for the closed loop system to have unity gain with characteristic polynomial:

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} \tag{10}$$

Analytical calculation of the closed loop characteristic polynomial using the control action (3) and comparison to (10) yields the following feedback gain vector, which is a function of the lagged output variable:

$$\boldsymbol{k}_k = \begin{bmatrix} \dfrac{0.7 - d_2}{y_{k-1}} & -\dfrac{1 + d_1 + d_2}{y_{k-1}} \end{bmatrix}$$

Here, it is evident that y_{k-1} needs to be non–zero. Examining the difference equation (9), it is clear that the output at sampling instant $k+1$ is not dependent on u_k if $y_{k-1} = 0$, which means that the system is not controllable. The same conclusion can be reached by calculating the frozen system (linear) controllability matrix or, in general nonlinear form, the SDP/PIP pole assignability conditions: see Taylor et al. (2008) for details. Therefore, the constraint $y_{k-1} \neq 0$ should be imposed before the calculation of $\boldsymbol{k}_k$.

Note that this type of problem is unlikely to arise for a practical application, since such a singularity would not be expected to lie in the normal operating range of an engineering device (or else the system would be uncontrollable). In practice, there would be a physical reason for this uncontrollable region of the state. Nonetheless, the present artificial example has been chosen to illustrate such a feasibility issue and how it can be identified.

In order to highlight the differences between a closed loop system with and without a RG, a deadbeat (i.e. $d_1 = d_2 = 0$) SDP/PIP design is selected. Starting from the initial condition $(y_0, u_0, z_0) = (1, 0.3, 0)$, two steps in the reference input are presented to the system. A positive step from 1 to 2 units after 9 samples and a negative step from 2 to 1 units after 40 samples.

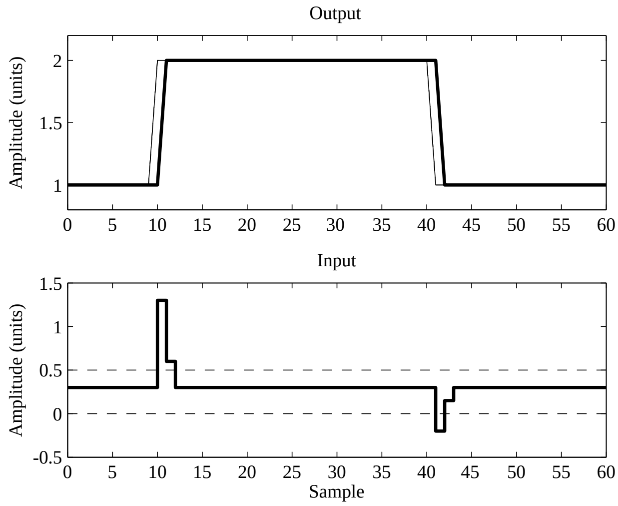

Fig. 1. Closed loop simulation when the constraints are not imposed on the system. The reference signal (thin line) and the system output (thick line) are shown in the upper subplot. The lower subplot shows the control signal (solid line) and the desired constraints (dashed lines).

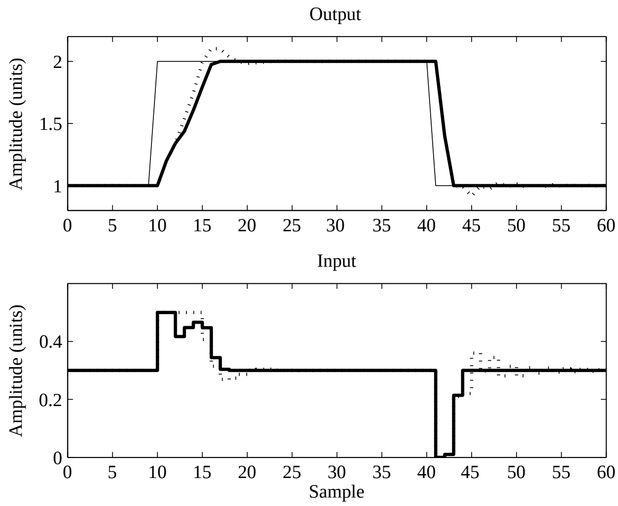

Fig. 2. Response (upper subplot) and control signal (lower subplot) of the SDP/PIP scheme with (thick solid line) and without (dotted line) a RG, both determined in the presence of constraints. The externally specified reference signal (thin solid line) is also shown.

It is further assumed that the control input should be non–negative, less that 0.5 units (i.e. $0 \leq u_k \leq 0.5$) and that it saturates at these levels.

Fig. 1 presents the system output and control signal assuming absence of the constraints. As expected given the selected poles, the closed-loop response of the nonlinear system follows a deadbeat trajectory. However, to achieve such a response the control signal clearly violates the constraints.

Next, the saturating constraints are imposed on the system and the SDP/PIP closed loop responses with (based on a prediction horizon of $N_p = 3$) and without the RG is depicted in Fig. 2. Here, it should be noted that in order to avoid integral windup the incremental form of the SDP/PIP controller is utilised, as described by (5). From Figure 2, it could be argued that in the

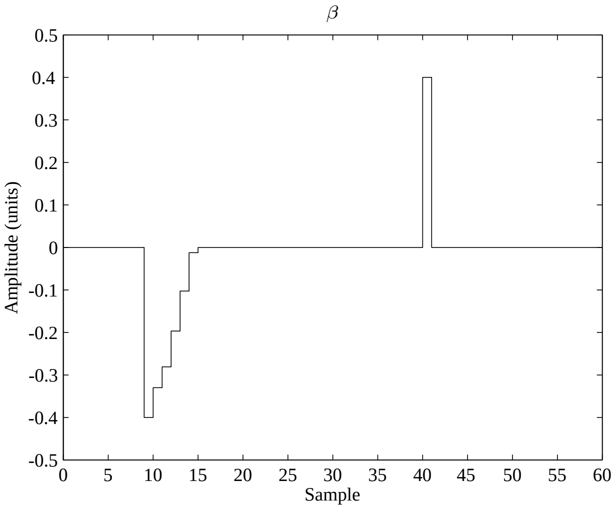

Fig. 3. The amplitude of the β parameter for the RG simulation in Figure 2.

absence of the RG the closed loop system has a faster rise time, makes more use of the available control signal and therefore resembles more closely the required deadbeat response. However, use of the RG avoids an undesirable overshoot of the reference. Furthermore, during the negative step, the settling time of the system without reference management is considerably longer. Only by using the RG in this way, is the dead-beat requirement presented to the design equations at each sampling instant.

Finally, Fig. 3 shows the evolution of the β–parameter for the case of the RG closed-loop response. The amplitude of this parameter provides an indication of the effect of the constraints on the closed loop system. It this case, for the first step in the reference input r_k, the RG brings the adjusted reference w_k much closer to actual output and subsequently moves it back towards the desired output in small steps. This is because the available control action does not allow for a faster response. During the second step, the similar behaviour of the RG is initially observed in reverse. However, in this case the available control action requires only one correction back to $r_k = w_k$. The reason for this asymmetry is because the steady state value of the control input is further from the constraint in the direction required for this negative step.

This section illustrated the constraint handling technique proposed in Section 3 for SDP systems. However, it should be noted that further investigations are required for a complete overview of constraint handling of non-linear systems described within the SDP framework. As for any non-linear system, different initial conditions may affect the response of the closed-loop system which is non considered here. Finally, the evaluation of the proposed technique in a practical application is the focus of ongoing research by the authors and will be presented in future publications.

5. CONCLUSIONS

This paper has developed a Proportional–Integral–Plus (PIP), Reference Governor (RG) approach for the control of nonlinear systems described by State Dependent Parameter (SDP) models. A low level stabilising SDP/PIP controller steers the system in the desired direction and a Reference Governor (RG) accounts for constraints in the system variables. This contrasts with the off-line simulation-based methods previously used for SDP/PIP design. The degrees of freedom of the RG are parametrised in a way that closely resembles the Closed Loop Paradigm of Rossiter (2003). This allows for a transparent observation and quantification of the effect of constraints on the control system.

Preliminary simulation experiments, such as the example discussed in the present paper, suggest that the addition of the RG can improve the reliability of SDP/PIP control systems in the presence of constraints. Since such SPD/PIP control systems have already been shown to perform well in a range of practical applications (Stables and Taylor, 2006; Taylor et al., 2007b), it is hoped that the proposed technique will encourage further applications in situations requiring on-line handling of system constraints.

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