A multiple-observer approach to stability in wireless network control systems

Adrian McKernan * **Carlos Ariño** ** **George W. Irwin** *
William G. Scanlon * **Jian. Chen** *

* *Intelligent Systems and Control group, School of Electronics, Electrical Engineering and Computer Science Queens University Belfast, UK (e-mail: amckernan01@qub.ac.uk, g.irwin@ee.qub.ac.uk and w.scanlon@qub.ac.uk)*
** *Department of Industrial Systems Engineering and Design, Universitat Jaume I, Avenida Vicent Sos Baynat, Castell de la Plana, Spain (Email: arino@esid.uji.es).*

Abstract:
This paper describes a new multiple-observer approach to Wireless Network Control Systems (WNCS). Two sets of observers are proposed, Lost Sample Observers (LSO) to deal with packet dropout and State Prediction Observers (SPO) to compensate for time-varying delays. These are designed using Linear Matrix Inequalities (LMI), thereby ensuring closed-loop stability. A numerical example, of a cart-mounted inverted pendulum is given along with results from simulation studies, comparing this new approach with a constant gain Linear Quadratic Regulator (LQR), in the presence of time-varying delays. Practical experimental results, on a IEEE 802.11b wireless channel in a reverberation chamber, further confirm the efficacy of the approach.

Keywords: Networked control system (NCS), LMI, Varying time delay, Stability

1. INTRODUCTION

Network Control Systems (NCS), using TCP/IP protocols have been an area of considerable research activity because of the practical benefits and the theoretical issues arising from the use of a communication channel in a feedback system. Wireless networks are even more challenging for feedback control since, as well varying delays due to traffic congestion and collision on the network, the wireless channel itself also presents a problematical data transmission path.

The NCS literature ranges from the design of new communications protocols for reducing delays (Walsh et al., 2002) to strategies for coping with them. For example, an early use of observers to compensate for NCS delays was proposed in Luck and Ray (1990), however this only deals with constant delays. Liu et al. (2007) discusses the stability of a networked predictive controller. A reduced communication strategy involving adaptive sampling rates was proposed for wireless networked control in Colandairaj et al. (2007b) and the benefits demonstrated in Colandairaj et al. (2007a), with sampling based on a communications Quality of Service measurement. Event based control is explored in Cervin and Åström (2007) and Johannesson et al. (2007) with sampling based on a control performance indicator. In Sala (2005), LMIs were used to design both a non-stationary observer and feedback gains for systems with time-varying sampling and delays. Penarrocha et al. (Dec. 2005) considered the issues of scarce measurements (lost samples) and delays while Sanchis et al. (2007) described a predictor for getting predicted states at regular intervals between sampling instances.

The majority of the work to date has concentrated on wired NCS with rather less attention given to wireless networks. This paper addresses the problems of both time varying delays and packet drop out for a WNCS. A set of LSO observers are used to compensate for varying numbers of sequential packet drops and future states are estimated using SPO observers. By invoking the Separation Principle the design of the feedback gain and observers have been separated into LQR and LMI problems. With a stable control law, overall stability has been proved by designing observers where the state estimation error tends to zero using LMIs. Experimental results of a cart-mounted inverted pendulum controlled across a IEEE 802.11b wireless channel in a reverberation chamber are included. These results, derived under controllable multipath conditions represent an important and unique contribution, allowing for a meaningful comparison of the new multi-observer approach with a constant gain LQR solution. Further the recording of the Round Trip Delay (RTD) introduced by the wireless network have facilitated realistic simulation studies on the new multi-observer approach.

This paper will be organised as follows the next section will discusses preliminary definitions and notation. Section 3 outlines WNCS and describes the new multi-observer approach. The theoretical proof of stability using LMIs is presented in section 4 followed by the numerical example in section 5. Sections 6 and 7 contain the results of simulation studies and the IEEE 802.11b experiments respectively. The paper ends with brief conclusions.

2. PRELIMINARIES

The problem of time-varying delayed measurements can be modelled as a time-varying discrete system. This is true for the case of linear state-space control over a wireless network. Consider a nth-order Linear Time-Invariant (LTI) continuous-time process:

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

with unknown time-varying delays between sampling and actuation $\tau_{sa} = 0, \tau_1, \tau_2, \ldots, \tau_{N_1}$.

Suppose the system is sampled at time instants $t_k \in \mathbb{R}$, $k = 0, 1, \ldots$. The time-varying sampling period will then be $T_k = t_{k+1} - t_k$. If a Zero Order Hold device (ZOH) is used to keep the input constant during the sampling interval the state at t_{k+1} is given by

$$x(t_{k+1}) = e^{A(t_k)}x(t_k) + \int_0^{t_k} e^{A(t_k-\tau)} B d\tau u(t_k) \tag{2}$$

Introducing the notation $A(t) = e^{At}$, $B(t) = \int_0^t e^{A(t-\tau)} B d\tau$, $A_k = A(t_k)$, $B_k = B(t_k)$, $x_k = x(t_k)$, $u_k = u(t_k)$ then produces

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k \\ y_k &= C_k x_k \end{aligned} \tag{3}$$

Equation (3) constitutes a time-varying discrete time representation of the continuous linear system in (1). Given a minimal sample time T_s, then the system matrices A_k, B_k and C_k can be selected from a set of candidate ones A^j, B^j and C^j, where $j = T_s, 2T_s, \ldots, N_2 T_s$. This produces

$$\begin{aligned} x_{k+1} &= A_k^j x_k + B_k^j u_k \\ y_k &= C_k^j x_k \end{aligned} \tag{4}$$

3. PROBLEM OUTLINE

Consider the WNCS setup in Fig. 1 where the sensor and actuator are co-located with a remote controller connected by a wireless channel. The delay between sampling and actuation τ_{sa} consist of, the delays in the feedforward τ_{sc} and feedback τ_{ca} paths respectively, along with the control computation time τ_c . These delays combine to give the overall RTD $\tau_{sa} = \tau_{sc} + \tau_c + \tau_{ca}$.

The variation in these transmission delays are not caused by the time taken for the data to propagate over the network but rather by the time the stations must wait for the wireless channel to be free so they can to transmit (the contention time). If data is not received these waiting times are further increased by the doubling of the contention window for every retransmission (Gast, 2005). The fact that the data payload in control packets is small compared

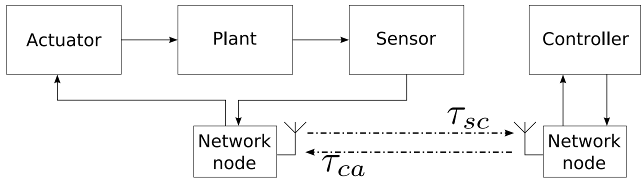

Fig. 1. Wireless Network Control System

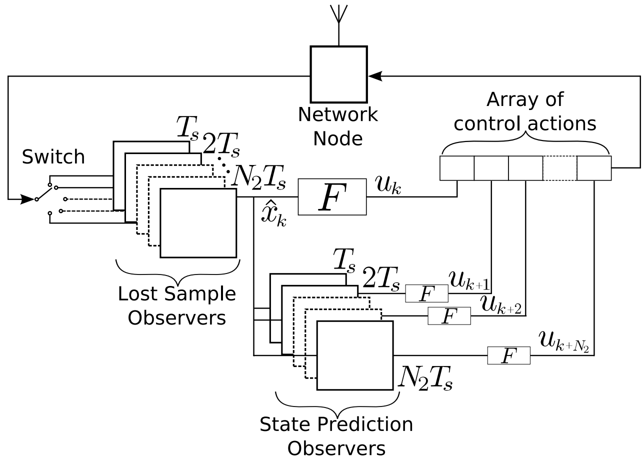

Fig. 2. Multi-observer based control for WNCS

to the overheads in a network frame means that any consequential extra propagation time is negligible compared to the overheads of contention and packetisation. This leads to the idea of exploiting this extra resource to improve the overall control performance (Liu et al., 2005) by including possible future control actions in any packet sent to the actuator. The actuator will then be able to select the control action closest to the time at which that packet arrives.

However this strategy requires timestamped packets and synchronous action between the sensor, controller and actuator. If, as in Fig. 1, the sensor and actuator are co-located τ_{sa} the RTD is available as the actuator has access to the same clock as the sensor. The controller then only needs to calculate the time between measurements by simply comparing subsequent timestamps. Co-location of the sensor and actuator means that the measurement packet can also contain current control action u_k.

3.1 Observer design

Fig. 2 illustrates the approach proposed here. This employs two separate sets of observers, one to compensate for the RTD, another to handle packet drop out.

The Lost Sample Observers (LSO) compensate for lost samples due to measurement packet drop out. Knowing the timestamp of the previous sample time $t_{k-(s+1)}$ where s is the number of dropped packets, and the current sample time t_k, the controller can select the observer with the correct sample period $T_k = t_k - t_{k-(s+1)}$ to produce the current estimate $\hat{x}_k$. The State Prediction Observers (SPO) estimate the future plant states by inputting $\hat{x}_k$ into a set of parallel observers each corresponding to a discrete system from (4) with a differing sample time to produce $\hat{x}_{k+1|k}, \hat{x}_{k+2|k}, \ldots, \hat{x}_{k+N_2|k}$. These predicted states are used to calculate the control actions.

3.2 Controller design

The design of the multiple-observer system and the design of the control gains has been separated into two distinct problems. The controller is designed as a constant gain feedback controller (5) in the form of a Linear-Quadratic Regulator (LQR). The LQR gains are designed for the continuous time linear system (1) without delays or packet drop out.

$$u_t = -Fx_t \tag{5}$$

In (5) F is the constant gain of the state feedback controller. Using the LQR gains for a continuous time system in the multiple-observer approach (Fig. 2) allows a comparison between the two approaches (§6). Note that the multiple-observer based WNCS can handle incomplete state information as well as allowing for the estimation of future states. The overall system with multiple-observers is giving in (6).

$$\begin{gathered} x_{k+1} = A_k^j x_k + B_k^j u_k \\ y_k = C_k^j x_k \\ \hat{x}_k = A_k^j \hat{x}_{k-1} + B_k^j u_{k-1} - L_k^j (C_k^j \hat{x}_{k-1} - y_{k-1}) \\ u_k = -F\hat{x}_k \end{gathered} \tag{6}$$

$\hat{x}_k$ are the predicted states of the plant and L_k^j are the observer gains at sample instance t_k.

4. STABILITY DESIGN

In Sala (2005), a Lemma was presented for the stability of time-varying sampled systems. It states that by the Separation Principle a system is stable if both the observer and controller are stable.

The stability of the observer can be proved by showing that the estimation error tends towards zero. From (6) this error is given by

$$\begin{aligned} e_k &= \hat{x}_k - x_k \\ \hat{x}_k - x_k &= A_k^j(\hat{x}_{k-1} - x_{k-1}) - L_k^j C_k^j (\hat{x}_{k-1} - x_{k-1}) \\ e_k &= A_k^j e_{k-1} - L_k^j C_k^j e_{k-1} \end{aligned} \tag{7}$$

Now, assuming the following candidate Lyapunov function

$$V_k = e_k^T P e_k \tag{8}$$

with the difference ΔV given by

$$\begin{aligned} \Delta V &= V_{k+1}(x) - V_k(x) \\ &= e_{k+1}^T P e_{k+1} - e_k^T P e_k \end{aligned} \tag{9}$$

Substituting (7) into (9) gives

$$\begin{aligned} \Delta V &= (A_k^j e_k - L_k^j C_k^j e_k)^T P (A_k^j e_k - L_k^j C_k^j e_k) - e_k^T P e_k \\ \Delta V &= e_k^T [(A_k^j - L_k^j C_k^j)^T P (A_k^j - L_k^j C_k^j) - P] e_k \end{aligned} \tag{10}$$

For stability the two inequalities (11) and (12) must hold.

$$e_k^T P e_k > 0 \tag{11}$$

$$e_k^T [(A_k^j - L_k^j C_k^j)^T P (A_k^j - L_k^j C_k^j) - P] e_k < 0 \tag{12}$$

If $P > 0$ then $e_k^T P e_k > 0$ is true for any scalar e_k. It follows that if $(A_k^j - L_k^j C_k^j)^T P (A_k^j - L_k^j C_k^j) - P < 0$ then (12) is also true, leaving the following two inequalities (13) and (14) which must hold for stability.

$$(A_k^j - L_k^j C_k^j)^T P (A_k^j - L_k^j C_k^j) - P < 0 \tag{13}$$

$$P > 0 \tag{14}$$

Taking the Shur complement of (13) (Boyd et al., 1994) produces

$$\begin{bmatrix} P & (PA_k^j - PL_k^j C_k^j)^T \\ PA_k^j - PL_k^j C_k^j & P \end{bmatrix} > 0 \tag{15}$$

and introducing the dummy variable $M_k^j = PL_k^j$, produces the LMI

$$\begin{bmatrix} P & (PA_k - M_k^j C_k^j)^T \\ PA_k^j - M_k^j C_k^j & P \end{bmatrix} > 0 \tag{16}$$

The overall system is stable if matrices $P = P^T$ and M_k^j can be found such that (16) and (14) are satisfied.

The observer gains can subsequently be obtained from

$$L_k^j = P^{-1} M_k^j \tag{17}$$

5. NUMERICAL EXAMPLE

Control of a cart-mounted inverted pendulum was chosen to demonstrate the multiple-observer approach for WNCS based state-feedback control as a suitable problem as this forms an open loop unstable system. The state model is given in (18).

$$\begin{gathered} \dot{x} = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{gathered} \tag{18}$$

where the A , B and C matrices are given by (19).

$$\begin{gathered} A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{p} & \frac{m^2gl^2}{p} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{p} & \frac{mgl(M+m)}{p} & 0 \end{bmatrix} \\ B = \begin{bmatrix} 0 \\ \frac{(I+ml^2)}{p} \\ 0 \\ \frac{ml}{p} \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{gathered} \tag{19}$$

The physical parameters given in Table 1 produce the continuous representation (20)

Symbol	Value	Value
M	Mass of cart	0.3kg
m	Mass of pendulum	0.1kg
b	Friction of cart	$0.1Nms^{-1}$
l	Length of Pendulum	0.7m
I	Inertia of Pendulum	$0.001kgm^2$
g	Acceleration due to gravity	$9.8ms^{-2}$

Table 1. Physical properties of inverted pendulum

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.182 & 2.673 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.455 & 31.182 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1.818 \\ 0 \\ 4.545 \end{bmatrix} u \tag{20}$$

The continuous-time LQR gains for this linear system which meets the plant performance specification in Table 2 was designed with $Q = diag([1000\ 0\ 100\ 0])$ and $R = 1$, giving the constant feedback gains in matrix F.

$$F = [-31.6 \quad -21.8 \quad 77.1 \quad 20.4] \tag{21}$$

A sample interval of $T_s = 0.01s$ produces the discrete-time state model (22).

$$x_{k+1} = \begin{bmatrix} 1.00 & 0.0100 & 0.0002 & 0 \\ 0 & 0.997 & 0.0318 & 0.0002 \\ 0 & -0.000 & 1.00 & 0.0100 \\ 0 & -0.0046 & 0.182 & 1.00 \end{bmatrix} x_k + \begin{bmatrix} 0.0002 \\ 0.0331 \\ 0.0002 \\ 0.0463 \end{bmatrix} u_k \tag{22}$$

Consider now a measurement packet y_k is received at time instant t_k. The matrices $A_k^{\tau_{sa}}$, $B_k^{\tau_{sa}}$, $C_k^{\tau_{sa}}$ and $L_k^{\tau_{sa}}$ can be used to predict the states $\hat{x}_{k+\tau_{sa}|k}$ but τ_{sa} the RTD is unknown as it is uncertain when the control packet will arrive at the actuator. The SPO sets of observers in Fig. 2 are used to estimate $\hat{x}_{k+\tau_{sa}|k}$ for different assumed values of τ_{sa}. These observers are designed off-line using LMIs to ensure stability. For example $\tau_{sa} = T_s = 0.01s$, the matrix $A^{0.01}$ in (22) and $C^{0.01}$ in (19) are used in (16) to calculate $L^{0.01}$ (23). The required control follows from $u_{k+0.01} = F\hat{x}_{k+1|k}$ with F defined in (21).

$$L^{0.01} = \begin{bmatrix} 1.79 & -0.00336 \\ 54.6 & -0.328 \\ -0.00733 & 1.80 \\ -0.758 & 55.9 \end{bmatrix} \tag{23}$$

This calculation of observer gains is repeated for various differing values of τ_{sa}. Each of the resultant SPO observers is applied when a measurement packet is received, thus creating the required array of possible control actions.

6. RESULTS

A simulation study was carried out with a varying delay τ_{sa} but with no packet loss, the delay having been recorded experimentally, as described in §7. The delay was inserted in the feedback path as the controller performs the same actions on receipt of the each measurement packet as there is no packet drop out. The actuator has a synchronised clock and calculates τ_{sa} (with a resolution of $0.5ms$) and selects the appropriate control action from the predicted sequence produced by the controller.

In this study the controller had three SPO observers designed with sampling times of $T_s = 10ms, 20ms, 30ms$ respectively to produce the array of control actions

$$[\, u_k \ u_{k+0.01} \ u_{k+0.02} \ u_{k+0.03} \,] \tag{24}$$

Representative graphical results comparing the performances of the LQR and proposed multiple-observer approaches to control for good (Figures 3, 4 and 5) and those for poor (Figures 3, 6 and 7) channel conditions

When the channel conditions are good there is little difference between the closed-loop performance of the multiple-observer and that of the constant gain LQR approaches (figures 4 and 5). Both approaches meet all the performance specifications (table 2).

As the wireless channel deteriorated the probability of much larger delays increased. In this case of poor channel conditions, the multiple-observer approach did not produce the desired specification in overshoot. However in contrast to the LQR approach it did manage to maintain stability, sustaining a pendulum angle (Fig. 7) of less than 1 radian as required (Table 2).

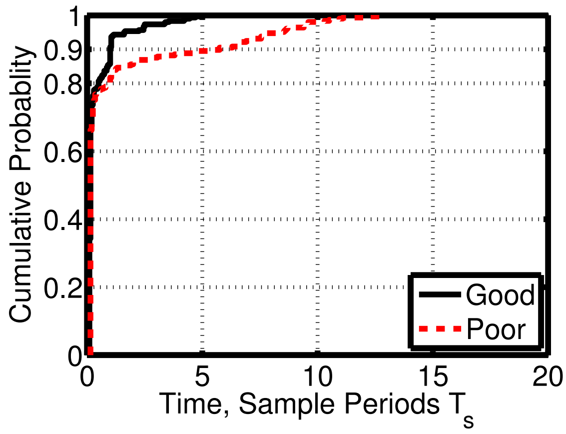

Fig. 3. Cumulative distribution functions for experimentally measured τ_{sa} variable time delay. Good channel conditions 90% of delays below T_s, poor channel conditions 90% of delays below $6T_s$

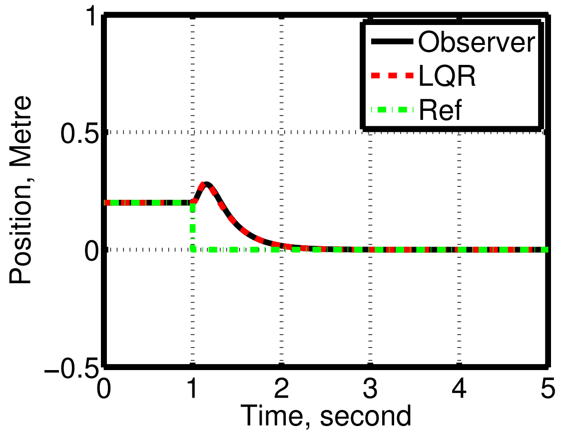

Fig. 4. Comparison of step response of cart for LQR and multiple-observer based control under good channel conditions

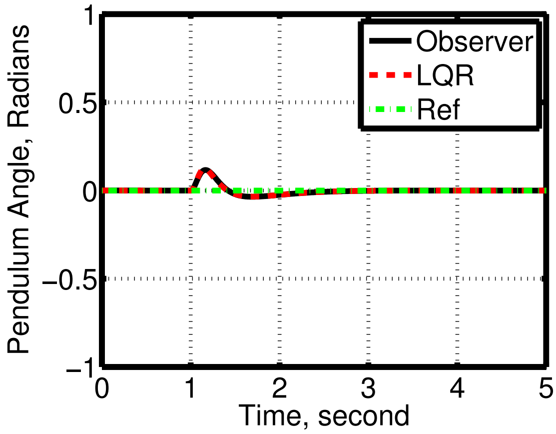

Fig. 5. Comparison of step response of pendulum angle for LQR and multiple-observer based control under good channel conditions

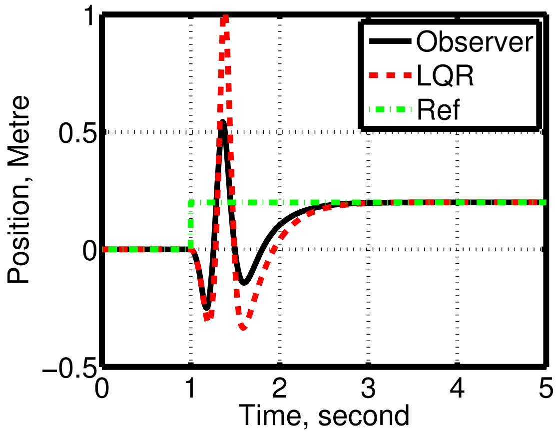

Fig. 6. Comparison of step response of cart for LQR and multiple-observer based control under poor channel conditions

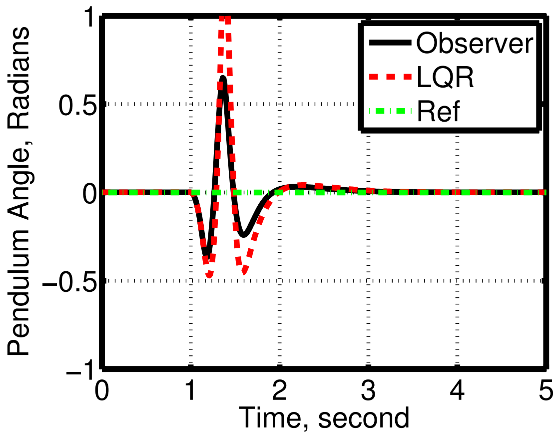

Fig. 7. Comparison of pendulum angle during step response for LQR and multiple-observer based control under poor channel conditions

7. EXPERIMENTAL WNCS RESULTS

An experimental testbed was created with two laptops, both running a 2.6.17 Linux kernel patched with the Real Time Application Interface (Bucher and Balemi, 2006).

Specification		Good		Poor	
		LMI	LQR	LMI	LQR
Settling Time x	$5s$	✓	✓	✓	✓
Rise Time x	$1s$	✓	✓	x	x
Overshoot desired θ	0.2 rad	✓	✓	x	x
Overshoot stable θ	1 rad	✓	✓	✓	x

Table 2. Performance comparisons of simulation studies

Specification		Good		bad	
		LMI	LQR	LMI	LQR
Settling Time x	$5s$	✓	✓	x	x
Rise Time x	$1s$	✓	✓	-	-
Overshoot desired θ	0.2 rad	✓	✓	x	x
Overshoot stable θ	1 rad	✓	✓	✓	x

Table 3. Performance comparison of experimental results

One laptop runs a real-time model of a cart-mounted inverted pendulum (19), while the other contains the controller (Fig. 8). This approach was chosen as it allowed for a real-time simulation with the added benefit of access to Linux networking. Communication between the remote PCs was done across a physical IEEE 802.11b wireless channel. Information was sent in User Datagram Protocol (UDP) datagrams, as UDP is a connectionless protocol. A packet capture program was used to record packet loss and monitor the condition of the network during experimental studies.

A reverberation chamber was employed (Fig. 8) to produce the multipath wireless conditions. This constitutes an over-moded resonant cavity and multipath environment, with computer controlled stirrers being able to change the modes, allowing WNCS experiments to be repeated under precisely controlled conditions. Note that compared to the simulation studies in §6 the use of a physical IEE802.11b channel causes both packet loss and delays.

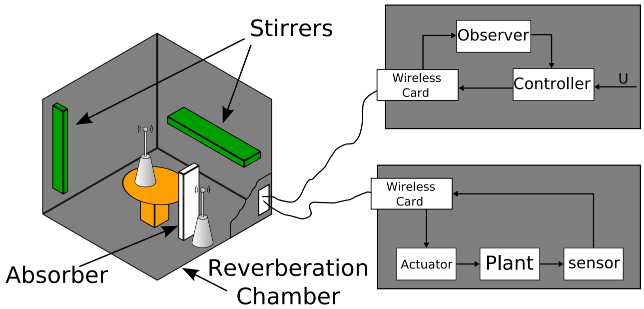

Fig. 8. Experimental setup

Representative results of the practical WNCS experiments will be shown to compare the LMI designed multiple-observer approach with the LQR controller. With good wireless channel conditions both controllers again meet the desired performance specification (Table 3). However for bad channel conditions while neither controller meets the settling time or overshoot specifications, the multiple-observer based control once again successfully maintains the pendulum angle (Fig. 11) within the required stability region of less than 1 radian (Table 3).

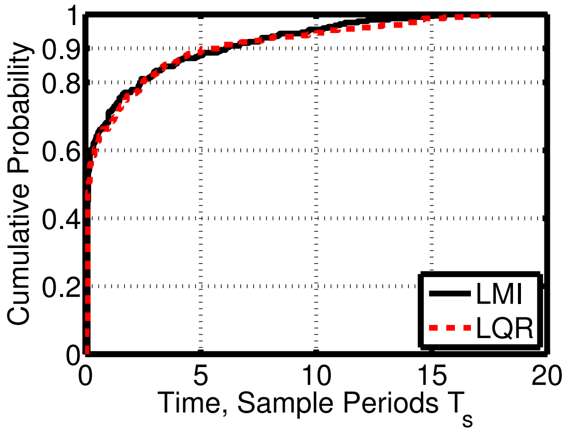

Fig. 9. Cumulative Distribution Functions for delays in WNCS experiments with multiple-observer and LQR based control - poor channel conditions (90% of Delays below $7T_s$)

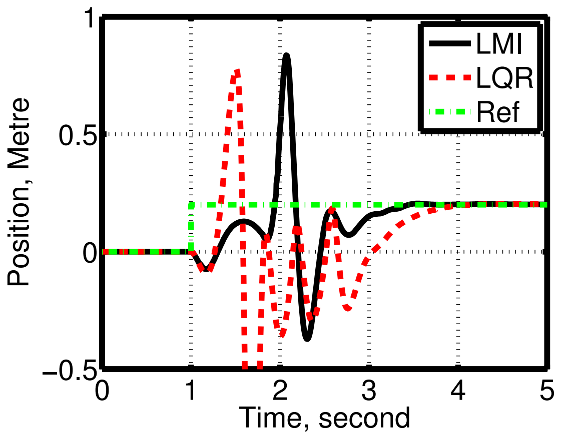

Fig. 10. Comparison of step response of cart for LQR and multiple-observer based control WNCS experiments under poor channel conditions

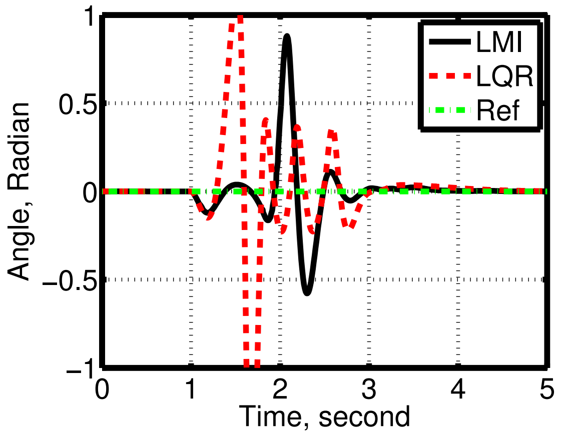

Fig. 11. Comparison of pendulum angle during step response for LQR and multiple-observer based control WNCS experiments under poor channel conditions

8. CONCLUSIONS

In this paper a new approach to WNCS has been proposed which involves the use of multiple-observers to deal with the problems of packet dropout and delay. Two sets of observers are proposed SPOs and LSOs, designed using using LMIs, thereby ensuring closed-loop stability. Simulation studies, comparing this new approach with LQR control in the presence of time varying delays, have shown that the new multiple-observer approach maintains stability in the presence of severe time-varying delays. This conclusion was also confirmed through additional experimental results involving a IEEE 802.11b wireless channel in a reverberation chamber.

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