Pole Placement Controller Design for Linear Parameter Varying Plants

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Abstract: Nonlinear plants can often be modelled as linear parameter varying (LPV) plants for which a number of techniques exist for control synthesis. However, there are some systems for which such a technique presents difficulties. In this paper, we consider one such system for which we propose a pole placement method using state feedback in order to cancel parameters variation of LPV plants. Hence, any linear time invariant (LTI) controller can subsequently be employed for an outer loop. The approach is demonstrated on an example for which only a single output can be measured. Therefore, a state observer for an LPV plant is also demonstrated in order to estimate state values. The simulation results show that the new approach yields reliable closed-loop stability with good closed-loop transient performance of the system.

1. INTRODUCTION

Although most real plants are nonlinear, they can often be modelled as linear parameter varying (LPV) plants model whereby their dynamic characteristics vary, following some time-varying parameters (Shamma and Athans, 1991). The time-varying parameters' values are unknown *a priori* but can be measured in real-time and lie in some set bounded by known minimum and maximum possible values.

A number of methods have been developed to design controllers for such LPV plants. Shamma and Athans (1991) first presented and analysed LPV plants with a gain scheduling approach that guarantees the stability, robustness and performance properties of the closed-loop system. Gahinet, *et al*. (1994) proposed the parameter-dependent Lyapunov function test for robust stability/performance of plants with fixed or time-varying uncertain parameters. This Linear Matrix Inequalities (LMI-based) test is always less conservative than the quadratic stability test (Gahinet, *et al*., 1994). Furthermore, Apkarian *et al*. (1994) developed a design method with H_∞-like control for LPV plants where the synthesis problem involves solving a system of LMIs. Wu and Packard (1995) also developed an LQG control design for LPV plants with use of a quadratic integral cost function for the performance objective.

In this paper we explicitly consider the example of Leith and Leithead (1999). In this example it was shown that for an LPV plant model derived from the Jacobian, a common approach, an LPV controller synthesized using the method of Apkarian, *et al*. (1994) is unstable when applied to the original nonlinear plant (Leith and Leithead, 1999). More detail of this problem is presented in the next section, where an introduction to LPV systems is also presented. In section 3, the robust stability analysis of LPV plants is briefly summarized. In addition, both quadratic and parameter-dependent Lyapunov functions are applied to test the stability of Leith and Leithead's (1999) example. In section 4, a pole placement approach is proposed and demonstrated on the example. The nonlinear simulation results are presented and show stability with good performance. This paper concludes with some comments.

2. LPV SYSTEMS & PROBLEM STATEMENT

A general class of LPV plants can be written in polytopic LPV models as a state-space system of the form:

$$\dot{x} = A(\theta(t))x + B(\theta(t))u \tag{1}$$

$$y = C(\theta(t))x + D(\theta(t))u \tag{2}$$

where $A(\cdot),\ldots,D(\cdot)$ are known functions and depend affinely on the time-varying parameters, $\theta(t)$, that vary in a polytope Θ of vertices ($\theta_1, \theta_2,\ldots,\theta_n$; $n = 2^m$; m is the total number of time-varying parameters). Typically, the LPV model is either derived from a nonlinear equation model or determined from experimental measurement by parameter identification methods (Verdult, *et al*., 2004). In this paper, we only consider the first case. Furthermore, to derive an LPV model, we restrict ourselves to only three methods; Jacobian linearization (Kwatny and Chang, 1998), function substitution (Tan, 1997), and state transformation (Shamma and Cloutier, 1993). However, there are also other methods such as velocity-based linearization (Leith and Leithead, 1999) and Carleman linearization (Berkolaiko, 1998) that can be used to derive an LPV model.

Consider the nonlinear plant example taken from Leith and Leithead (1999)

$$\dot{x}_1(t) = -x_1(t) + r(t) \tag{3}$$

$$\dot{x}_2(t) = x_1(t) - |x_2(t)|x_2(t) - 10 \tag{4}$$

$$y(t) = x_2(t) \tag{5}$$

An LPV model derived by using standard Jacobian linearization is given by (Leith and Leithead, 1999)

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & -2\theta \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [r] \tag{6}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{7}$$

where $z_1(t) = x_1(t) - x_{1o}$ and $z_2(t) = x_2(t) - x_{2o}$. It is noted that x_{1o} and x_{2o} are at the equilibrium point (or trim point). The range of $\theta(t) = |z_2(t)|$ is arbitrarily defined from 0 to 10. An LPV controller can be determined by using the standard MATLAB LMI Toolbox function (Gahinet, *et al.*, 1995), `hinfgs`, with the criterion

$$\left\| \begin{matrix} W_1 S \\ W_2 K S \end{matrix} \right\|_\infty < 1 \tag{8}$$

where the weighting function W_1 and W_2, taken from Leith and Leithead (1999), are

$$W_1(s) = \frac{0.5}{s + 0.002} \tag{9}$$

$$W_2(s) = \frac{0.02s}{s + 1000} \tag{10}$$

The routine `hinfgs` computes an LPV controller that stabilizes the closed-loop system and minimizes the closed-loop quadratic H_∞ performance, γ, that ensures the L_2 gain of the system is bounded by γ along all possible parameter trajectories, $\theta(t)$. The LPV controller obtained for this LPV model is

$$\dot{x}_c = (\alpha_0 A_{c0} + \alpha_1 A_{c1}) x_c + B_c (y_{ref} - y) \tag{11}$$

$$r = C_c x_c \tag{12}$$

where $\alpha_0 = (10 - \theta) / 10$, $\alpha_1 = \theta / 10$, and

$$A_{c0} = \begin{bmatrix} 11.4933e+00 & 220.5506e+00 & 93.1838e+00 & 15.6255e+03 \\ -4.1441e+00 & -34.9702e+00 & -26.0301e+00 & 46.2968e+03 \\ -578.7503e-03 & -12.5446e+00 & -10.2185e+00 & 20.4856e+03 \\ 1.0769e+00 & 2.1290e+00 & 16.1633e+00 & -66.1258e+03 \end{bmatrix}$$

$$A_{c1} = \begin{bmatrix} 11.4904e+00 & 220.4540e+00 & 93.4033e+00 & 15.6255e+03 \\ -4.2406e+00 & -38.2060e+00 & -18.6740e+00 & 46.2966e+03 \\ -359.2582e-03 & -5.1885e+00 & -26.9418e+00 & 20.4861e+03 \\ 684.0884e-03 & -11.0360e+00 & 46.0926e+00 & -66.1267e+03 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -2.4197e-03 \\ -6.4566e-03 \\ -26.6143e-03 \\ -18.6908e+00 \end{bmatrix}$$

$$C_c = \begin{bmatrix} -12.7061e+00 & -222.1982e+00 & -94.5584e+00 & -12.8570e+03 \end{bmatrix}$$

In this paper, the matrices ($A_{c0}, \ldots, C_c$) obtained by `hinfgs` give an optimal quadratic random-mean-squares (RMS) performance, $\gamma = 0.1211$. The matrices are different from those presented in Leith and Leithead (1999). Also note that the LPV controller presented in Leith and Leithead (1999) is actually open-loop unstable. The controller presented above is open-loop stable but the simulation results that are presented in figure 1 still show the closed-loop instability problem described in Leith and Leithead (1999). The exact reasons for the closed-loop instability are still not certain. It can be seen that the closed-loop system is stable when the LPV controller is applied to the LPV plant model for a step response that change in demand from -3 units to 0 units. However, when the same LPV controller is applied to the original nonlinear plant, the nonlinear closed-loop system appears to be unstable.

LPV plant models that are derived by using different methods are also investigated. The function substitution-based LPV model taken from Shin (2002) is

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & -\theta \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u] \tag{13}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{14}$$

where $z_1(t) = x_1(t) - x_{1o}$, $z_2(t) = x_2(t) - x_{2o}$ and $u(t) = r(t) - r_o$. It is noted that x_{1o}, x_{2o} and r_o are at a trim point that is set as $(x_{1o}, x_{2o}, r_o) = (10, 0, 10)$. Using `hinfgs` with the same weighting function and same range of $\theta(t)$, the LPV controller for this LPV model is obtained with $\gamma = 0.09469$. The state transformation-based LPV model taken from Shin (2002) is

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -1-2\theta & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u] \tag{15}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{16}$$

where $z_1(t) = x_1(t) - |x_2(t)| x_2(t) - 10$, $z_2(t) = x_2(t)$ and $u(t) = r(t) - |x_2(t)| x_2(t) - 10$. It is noted that input $u(t)$ and state $z_1(t)$ are changed as the scheduling parameter, $x_2(t)$, varies. Using `hinfgs` with the same weighting function and same range of $\theta(t)$, the LPV controller for this LPV model is obtained with $\gamma = 0.08863$.

Figure 2 shows the simulation results of the function substitution-based and state transformation-based LPV controllers. The closed-loop instability does not occur. According to figures 1–2, we make an assumption that function substitution and state transformation methods give an LPV plant model that more accurately represents the nonlinear plant than the Jacobian linearization method.

3. ROBUST STABILITY ANALYSIS

Although `hinfgs` guarantees the closed-loop stability whilst giving optimal quadratic RMS performance γ, the closed-loop instability still arises for the example. Therefore, both quadratic and parameter-dependent Lyapunov functions are then necessary to validate this closed-loop system and obtain guarantees of stability and performance in the face of plant uncertainty.

3.1 Quadratic Lyapunov function

Following (Gahinet, *et al.*, 1994), such an LPV system is given as,

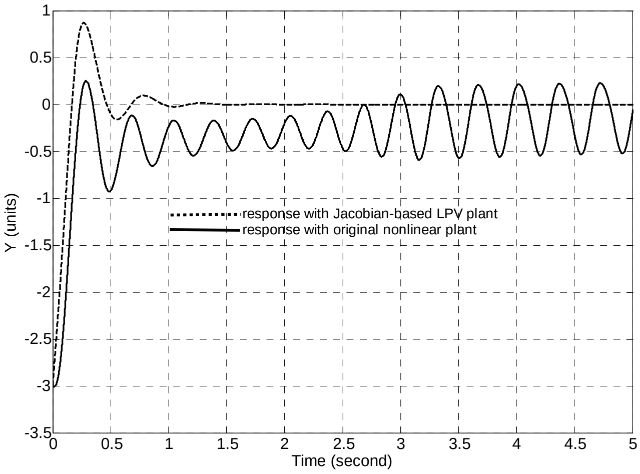

Fig. 1. Nonlinear step response from -3 to 0 of Jacobian-based LPV controller.

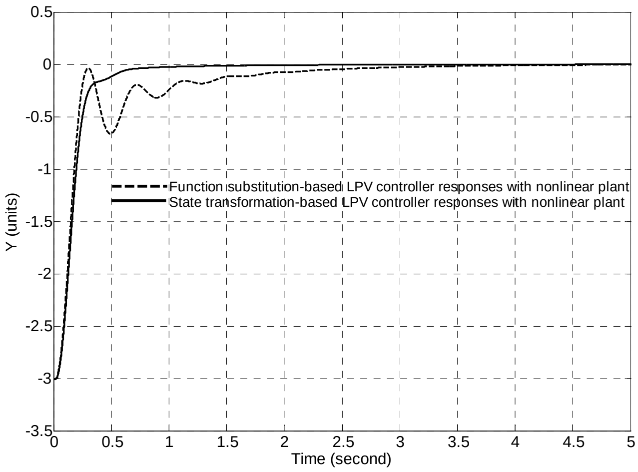

Fig. 2. Nonlinear step response from -3 to 0 of function substitution-based and state transformation-based LPV controllers.

$$\dot{x}(t) = A(\theta(t))x(t) \tag{17}$$

with $\theta(t)$ varying in a polytope Θ of vertices. The system is quadratically stable if there exists a positive-definite single Lyapunov matrix $P > 0$

$$V(x(t)) = x(t)^T Px(t) \tag{18}$$

such that

$$\frac{d}{dt}(V(x(t))) < 0 \tag{19}$$

along all state trajectories. This is equivalent to

$$A(\theta(t))^T P + PA(\theta(t)) < 0 \tag{20}$$

at all admissible values and trajectories of the time-varying parameters $\theta(t)$. Note that quadratic Lyapunov function guarantees stability over the entire parameter range and for arbitrarily fast parameter variations, which is their main source of conservatism. The quadratic stability of polytopic or affine parameter-dependent systems can be tested by using the MATLAB function `quadstab` (Gahinet, *et al*., 1995).

Having used function `slft` (Gahinet, *et al*., 1995), the closed-loop system of the Jacobian-based LPV controller and LPV plant model can be otained in polytopic model form. As a result of `quadstab`, this closed-loop system is quadratically stable giving the positive-definite Lyapunov matrix

$$P = \begin{bmatrix} 0.3700 & 0.0223 & 0.3692 & -0.0313 & 0.0177 & -0.0011 \\ 0.0223 & 0.0363 & 0.0214 & -0.0162 & 0.0206 & -0.0043 \\ 0.3692 & 0.0214 & 0.3687 & -0.0300 & 0.0179 & -0.0001 \\ -0.0313 & -0.0162 & -0.0300 & 0.0203 & -0.0040 & 0.0119 \\ 0.0177 & 0.0206 & 0.0179 & -0.0040 & 0.0233 & 0.0052 \\ -0.0011 & -0.0043 & -0.0001 & 0.0119 & 0.0052 & 0.1437 \end{bmatrix}$$

3.2 Parameter-Dependent Lyapunov function

Following Gahinet, *et al*., (1994), given an LPV system of the form of (17), the system is affinely quadratically stable if there exists a positive-definite Lyapunov matrix $P(\theta(t)) > 0$ with Lyapunov function

$$V(x(t),\theta(t)) = x(t)^T P(\theta(t))x(t) \tag{21}$$

where

$$P(\theta(t)) = P_0 + \theta_1 P_1 + \ldots + \theta_m P_m \tag{22}$$

such that

$$\frac{d}{dt}(V(x(t),\theta(t))) < 0 \tag{23}$$

along all state and admissible parameter trajectories. This is equivalent to

$$A(\theta(t))^T P(\theta(t)) + P(\theta(t))A(\theta(t)) + \frac{d}{dt}(P(\theta(t))) < 0 \tag{24}$$

at all admissible values and trajectories of the time-varying parameters $\theta(t)$ with interval bounds

$$\theta \in [\theta_{\min}, \theta_{\max}], \dot{\theta} \in [\nu_{\min}, \nu_{\max}] \tag{25}$$

Note that the parameter-dependent stability tests are always less conservative than the quadratic stability tests when the parameters are time-invariant or slowly varying (Gahinet, *et al*., 1994). The parameter-dependent stability of polytopic or affine parameter- dependent systems can be tested by using the MATLAB function `pdlstab` (Gahinet, *et al*., 1995). The result of `pdlstab` is that this Jacobian-based LPV closed-loop system is stable in the specified parameter range ($\theta(t) = |z_2(t)|$ varying from 0 to 10).

Hence `hinfgs`, `quadstab` and `pdlstab` only guarantee the closed-loop stability of an LPV controller and an LPV plant model but they could not guarantee the closed-loop stability of the LPV controller and the original nonlinear plant. For this particular example, we make another assumption that there is a mismatch between the Jacobian-based LPV model and the original nonlinear model. However, both function substitution-based and state transformation-based LPV models are identical to the original nonlinear model as shown below.

First, we show that the function substitution-based LPV model is identical to the original nonlinear model. By substituting

$$r(t) = u(t) + 10 \tag{26}$$

$$x_1(t) = z_1(t) + 10 \tag{27}$$

$$x_2(t) = z_2(t) \tag{28}$$

in (3) – (5). A new nonlinear equation can be obtained in the form.

$$\dot{z}_1(t) = -z_1(t) + u(t) \tag{29}$$

$$\dot{z}_2(t) = z_2(t) - |z_2(t)| z_2(t) \tag{30}$$

$$y(t) = z_2(t) \tag{31}$$

which can be rearranged as an LPV equation of the form (13) – (14).

Next, we show that the state transformation-based LPV model is identical to the original nonlinear model. By substituting

$$r(t) = u(t) + (|x_2(t)| x_2(t) + 10) \tag{32}$$

$$x_1(t) = z_1(t) + (|x_2(t)| x_2(t) + 10) \tag{33}$$

$$x_2(t) = z_2(t) \tag{34}$$

in (3) – (5). Another nonlinear equation can be obtained in the form.

$$\dot{z}_1(t) = -(1 + 2|z_2(t)|) z_1(t) + u(t) \tag{35}$$

$$\dot{z}_2(t) = z_1(t) \tag{36}$$

$$y(t) = z_2(t) \tag{37}$$

which can be rearranged as an LPV equation of the form (15) – (16).

Consider the transfer function of LPV plants (1) – (2) that is given by

$$G(s,\theta) = \{C(\theta)[sI - A(\theta)]^{-1} B(\theta)\} + D(\theta) \tag{38}$$

Substituting the matrices $A(\cdot),\ldots,D(\cdot)$ of the Jacobian-based LPV model (6) – (7) in (38), the transfer function can be determined as

$$G(s,\theta) = \frac{1}{(s+1)(s+2\theta)} \tag{39}$$

where $\theta = |z_2|$ varying from 0 to 10. This transfer function has two poles; one pole fixes at -1, the other pole varies from -20 to 0. The location of the varying pole of the Jacobian-based LPV model is equal to -2θ, but the true location of the varying pole of the original nonlinear plant is not equal to -2θ whenever this nonlinear plant is not in an equilibrium condition. As $y(t) = z_2(t)$ moves closer to 0, the mismatch uncertainty between the Jacobian-based LPV model and the original nonlinear model becomes more significant and makes the nonlinear closed-loop system unstable.

Having determined the reasons for the nonlinear plant closed-loop instability, the problem can be solved by simply increasing the conservativeness of the LPV plant model. That is, by setting a new range of the time-varying parameter to cover the uncertainty in the region close to right-half *s*-plane. For example, setting $\theta = |z_2|$ to vary from -1 to 10, indicates that the varying pole can vary from -20 to 2 even though, in fact, it can only vary from -20 to 0. Using `hinfgs` with the same weighting function as previously but the new range of θ, the new LPV controller is obtained with $\gamma = 0.1463$. The simulation results of the nonlinear closed-loop system with the new Jacobian-based LPV controller are presented in figure 3. The closed-loop instability disappears but the transient performance is degraded because of setting a more conservative range of θ.

4. POLE PLACEMENT APPROACH

Pole placement with state feedback can be used to overcome the closed-loop instability problem without degrading the transient performance. In this approach, we restrict ourselves to special LPV plants of the form

$$\dot{x} = A(\theta(t))x + Bu \tag{40}$$

$$A(\theta(t)) = A_0 + \theta_1(t)A_1 + \theta_2(t)A_2 + \ldots + \theta_m(t)A_m \tag{41}$$

$$y = Cx \tag{42}$$

where $A(\cdot)$ is known functions and depends affinely on time-varying parameters, $\theta(t)$. Furthermore, in order to apply state feedback and state observer, this LPV plant is assumed to be state controllable and observable for all possible parameters trajectories $\theta(t)$.

We apply state feedback $u = -K(\theta(t))x + n$. The state feedback gain, $K(\theta(t))$, is parameter-dependent and can be a nonlinear function of $\theta(t)$. Substituting u in (40), the state feedback closed-loop system becomes

$$\dot{x} = A^*(\theta(t))x + Bn \tag{43}$$

$$A^*(\theta(t)) = [A(\theta(t)) - BK(\theta(t))] \tag{44}$$

$$y = Cx \tag{45}$$

where n is the new input of the state feedback closed-loop system. By determining the state feedback gain-scheduling, it is possible to achieve any closed-loop eigenvalue assignment. However for the example, the states cannot be measured. Hence, a state observer is used to estimate state values. A general state observer can be constructed using observer feedback gain $K_e(\theta(t))$ which is parameter-dependent and can be a nonlinear function of $\theta(t)$. The state observer closed-loop system is given by

$$\dot{x}_e = [A(\theta(t)) - K_e(\theta(t))C]x_e + Bn + K_e(\theta(t))Cx \tag{46}$$

Subtracting (46) from (40), we obtain

$$\dot{x} - \dot{x}_e = [A(\theta(t)) - K_e(\theta(t))C](x - x_e) \tag{47}$$

To demonstrate the method, we consider the Jacobian-based LPV model (6) – (7) taken from Leith and Leithead (1999). We also select the state feedback closed-loop system to have

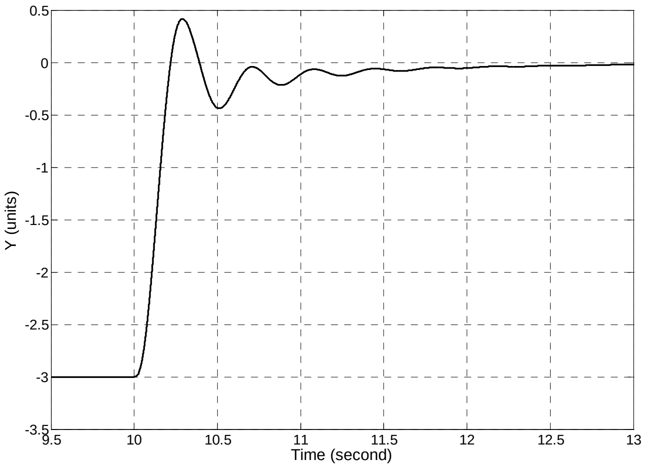

Fig. 3. Nonlinear step response from -3 to 0 of new Jacobian-based LPV controller with original nonlinear plant.

a realistic closed loop characteristic, i.e. natural frequency, $\omega_n = 10$ rad/s and damping ratio, $\zeta = 0.707$, in order to prevent actuator saturation. Hence, the desired characteristic equation can be written as

$$\lambda^2 + (2\times 0.707\times 10)\lambda + 10^2 = 0 \tag{48}$$

The state feedback gain-scheduling $K(\theta(t))$ can be determined by solving

$$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \left\{ \begin{bmatrix} -1 & 0 \\ 1 & -2\theta \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \right\} \right| = 0 \tag{49}$$

$$\lambda^2 + (2\theta + k_1 + 1)\lambda + (2\theta[1 + k_1] + k_2) = 0 \tag{50}$$

Equating coefficients of the polynomial yields the state feedback gain-scheduling as

$$k_1 = 13.14 - 2\theta, k_2 = 4\theta^2 - 28.28\theta + 100 \tag{51}$$

Having determined the state feedback gain-scheduling, the observer feedback gain-scheduling $K_e(\theta(t))$ can be determined by solving

$$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \left\{ \begin{bmatrix} -1 & 0 \\ 1 & -2\theta \end{bmatrix} - \begin{bmatrix} k_{e1} \\ k_{e2} \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \right\} \right| = 0 \tag{52}$$

$$\lambda^2 + (2\theta + k_{e2} + 1)\lambda + (2\theta + k_{e1} + k_{e2}) = 0 \tag{53}$$

The dynamics of the state observer must be faster than the system being controlled. Therefore we select the state observer to have a suitable characteristic, i.e. $\omega_n = 40$ rad/s and $\zeta = 0.707$, in order to avoid amplifying the noise of the controlled output. Then, the desired characteristic equation can be written as

$$\lambda^2 + (2\times 0.707\times 40)\lambda + 40^2 = 0 \tag{54}$$

Equating the coefficients of the polynomial yields the observer feedback gain-scheduling as

$$k_{e1} = 1543.44, k_{e2} = 55.56 - 2\theta \tag{55}$$

Substituting the matrices $A(\cdot)$, $C(\cdot)$ and $K_e(\cdot)$ of the state observer in (47) yields a state-space form as

$$\begin{bmatrix} \dot{z}_1 - \dot{z}_{e1} \\ \dot{z}_2 - \dot{z}_{e2} \end{bmatrix} = \begin{bmatrix} -1 & -1543.44 \\ 1 & -55.56 \end{bmatrix} \begin{bmatrix} z_1 - z_{e1} \\ z_1 - z_{e1} \end{bmatrix} \tag{56}$$

The state observer has two poles at $-28.3 \pm 28.3i$. Since both poles are in the left-half s-plane, the state observer is stable.

Having applied the state observer and the state feedback to the Jacobian-based LPV model, the state feedback closed-loop system can be determined by substituting the matrices $A(\cdot),\ldots,C(\cdot)$ and $K_e(\cdot)$ in (43) – (45) yielding the state-space form as

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 2\theta - 14.14 & -4\theta^2 + 28.28\theta - 100 \\ 1 & -2\theta \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [n] \tag{57}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{58}$$

where n is the new input of state feedback closed-loop system. Substituting the matrices $A(\cdot),\ldots,C(\cdot)$ of the state feedback closed-loop system into (38) gives the transfer function

$$G(s) = \frac{1}{s^2 + 14.14s + 100} \tag{59}$$

The state feedback closed-loop system has two constant poles at $-7.07 \pm 7.07i$. Both poles are in the left-half s-plane therefore the state feedback closed-loop system is stable.

According to the state feedback closed-loop transfer function, this approach shows the parameters variation of the special LPV plants can be cancelled. In order to obtain performance from the system, an additional linear time invariant (LTI) controller can be applied as an outer loop as shown in figure 6. For this particular example, an H_∞-mixed-sensitivity controller is used. With the same weighting function as previously, the controller obtained for the state feedback closed-loop system is

$$\dot{x}_c = A_c x_c + B_c (y_{ref} - y) \tag{59}$$

$$n = C_c x_c \tag{60}$$

The matrices ($A_c,\ldots,C_c$) are

$$A_c = \begin{bmatrix} -0.002 & 0 & 0 & 0 \\ 1355.3 & -61.186 & -14.911 & -38.782 \\ 84.705 & 58.676 & -15.072 & -14.924 \\ 0 & 0 & 8 & 1.2624e-015 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0.079057 \\ 0 \\ -1.944e-016 \\ 3.992e-016 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 2142.9 & 1484.4 & -23.577 & -61.319 \end{bmatrix}$$

The simulation results of the nonlinear closed-loop system are presented in figures 4 and 5. The closed-loop instability problem is solved with good transient performance of the system.

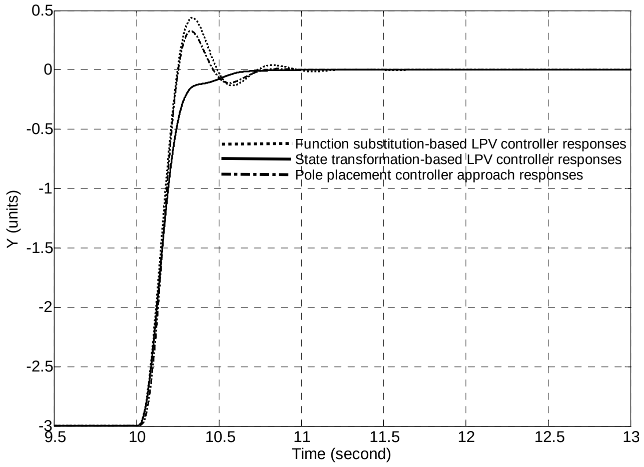

Fig. 4. Nonlinear step response from -3 to 0 with original nonlinear plant.

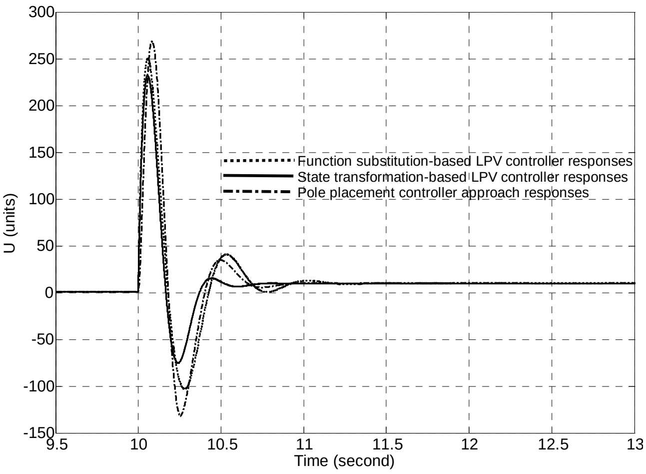

Fig. 5. Control input to original nonlinear plant.

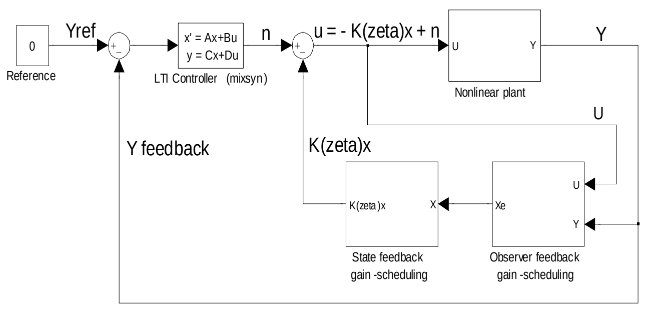

Fig. 6. Pole placement controller design.

5. CONCLUDING REMARKS

In this paper, a design method for cancelling the parameters variation of the example of Leith and Leithead (1999) by pole-placement state feedback is proposed. For the example, the approach yields reliable closed-loop stability and good closed-loop transient performance of the system because it makes the nonlinear plant appear to be an LTI plant, hence well-developed LTI tools can be applied. However, we emphasize that the approach is not applicable to a general class of LPV plants but it is applicable to only special LPV plants model of the form (40) – (42).

The example from Leith and Leithead (1999) is very interesting. The closed-loop instability occurs because the uncertainty between the Jacobian-based LPV model and the nonlinear model is in a region close to the right-half s-plane. The `hinfgs`, `quad-stab`, and `pdlstab` only guarantee the closed-loop stability of an LPV controller and an LPV model but they could not guarantee the closed-loop stability of the LPV controller and the nonlinear model.

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