Motion stabilization in the presence of friction and backlash: a hybrid system approach

Lőrinc Márton *,[1] **Béla Lantos** **

* *Dept. of Electrical Engineering, Sapientia Hungarian University of Transylvania, 540485 Targu Mures, Op. 9, Cp. 4, Romania. (e-mail: martonl@ms.sapientia.ro)*
** *Dept. of Control Engineering and Information Technology, Budapest University of Technology and Economics, H-1117 Budapest, Magyar Tudósok Körútja 2, Hungary. (e-mail: lantos@iit.bme.hu)*

Abstract: In this paper a hybrid system approach is considered to deal with backlash and friction induced nonlinearities in mechanical control systems. To describe the low velocity frictional behaviour a linearized friction model is proposed. The novelty of this study is that based on the introduced friction model, the stability theorems developed for hybrid systems can directly be applied for controller design of mechanical systems in the presence of Stribeck friction and backlash. During the controller design it is assumed that the size of the backlash gap is unknown and the load side position and velocity cannot be measured. For motion control an LQ controller is applied. A condition is formulated for the control law parameters to guarantee the asymptotic stability of the control system. Simulation measurements were performed to confirm the theoretical results.

Keywords: Motion Control, Friction, Backlash, Hybrid Systems, LQ control

1. INTRODUCTION

The friction and backlash are the most common non-smooth nonlinearities that may deteriorate the control performances in the mechanical control systems. The friction is present in every mechanical system in which the moving parts are in contact. The backlash appears mainly in gear transmissions where the moving parts temporarily loose the direct contact.

In many mechanical systems the effect of friction and backlash are overlapped. In these control systems a controller designed to compensate only the friction may perform poor in the presence of backlash and vice versa. For these systems both nonlinearities must be taken into consideration during controller design.

The nonlinear behavior of friction is accentuated in the low velocity regime. If fluid lubrication is applied, decreasing friction with increasing velocities can be expected in the low velocity regime (Stribeck phenomena). In the high velocity regime the friction force slowly increases with the velocity. To describe this phenomena the following model can be applied:

$$\tau_f = \begin{cases} \tau \quad for\ \dot{q} = 0\ and\ |\tau| \leq F_S \\ (F_C + (F_S - F_C)e^{-|\dot{q}|/\dot{q}_S})sign(\dot{q}) + F_V\dot{q}\ otherwise \end{cases} \quad (1)$$

where F_C is the Coulomb friction coefficient, F_S is the static friction coefficient, F_V is the viscous friction coefficient, $\dot{q}$ denotes the velocity (or angular velocity in the case of rotational motion), $\dot{q}_S$ is the Stribeck velocity, τ denotes the generalized tangential control force.

In the last years many dynamic models were introduced to describe precisely the friction phenomena de Wit et al. [1995], Swevers et al. [2000], Lampaert et al. [2002], Dupont et al. [2002]. However, experimental measurements have proved that a good static friction model can approximate the real friction force with a degree of 90% confidelity Armstrong-Hèlouvry [1991]. The dynamic friction behaviour can be introduced in the static model as a bounded additive model uncertainty de Wit et al. [1995]. It was also shown in Hensen et al. [2003] that the switching static friction model and the dynamic friction model predicts almost the same friction penomena induced limit cycles in controlled positioning systems. Hence the static friction model based compensation techniques still have great significance for practical applications. In a recent paper Putra et al. [2007] it was shown that the undercompensation of the friction force can lead to steady state error, while the overcompensation can cause limit cycles. It is why precise friction compensation is necessary in position control systems. Since the frictional parameters are slowly time varying, robust and adaptive control techniques are popular for friction compensation, see eg. Xie [2007], Makkar et al. [2007], Feemster et al. [1998].

The modeling and compensation of the backlash has also attracted a significant research effort over several decades.

[1] L. Márton is a grantee of the János Bolyai Scholarship

The recent survey paper Nordin and Gutman [2002] summarizes the introduced backlash models and compensation methods. Some papers focus in backlash induced limit cycles and unstable behavior, see e.g. Barreiro and Banos [2006]. In Tarbouriech and Prieur [2006] the stability of mechanical systems in the presence of backlash was studied based on Linear Matrix Inequalities. Other papers deal with the compensation of the backlash induced unstable behavior. Inverse backlash based compensation and its adaptive extension was proposed in Tao and Kokotovic [1996]. Hybrid model based Model Predictive Control (MPC) scheme for backlash compensation was introduced in Rostalski et al. [2007]. A linear estimator for fast and accurate estimation of the position and velocity in the presence of backlash in automotive power trains is described in Lagerberg and Egardt [2007].

In all of these studies dealing with backlash compensation the friction was omitted or very simple friction models were applied such as the viscous friction model. Surprisingly few papers deal with control of systems in the presence of backlash and nonlinear friction. The effect of backlash and stick-slip friction in heavy-duty hydraulic machines was studied in Nariman et al. [1996]. In the paper Menon and Krishnamurthy [1999] a two controller based switching control system was proposed to deal with low velocity friction and gear backlash. A soft computing approach for simultaneous Stribeck friction and backlash compensation was described in Suraneni et al. [2005].

The rest of the paper is organized as follows: In Section 2 a linearized hybrid model of the nonlinear controlled system is presented. In Section 3 the LQ controller design, based on the model introduced in the previous section, is described. Simulation results are shown in Section 4. Finally, Section 5 sums up the conclusions of this study.

2. MECHANICAL SYSTEM WITH FRICTION AND BACKLASH

To determine the dynamics of a one degree of freedom positioning system in the presence of friction and backlash, the so called inertia driven backlash model Nordin and Gutman [2002] is applied. In this model two regimes are separated: the *contact mode (CM)*, when the load is in contact with the motor shaft, the torque developed by the motor acts on the load. In *backlash mode (BM)*, that occurs when the direction of motion changes, there is no contact between the motor shaft and the load.

In backlash mode the dynamics is given by:

$$(BM)\begin{cases} J_M\ddot{q}_M = \tau - \tau_f \\ J_L\ddot{q}_L = 0 \end{cases} \tag{2}$$

The following notations were used: q_L is the load side position, q_M the load side position, J_L the inertia on the load side, J_M the inertia of the motor shaft, K_G the gear ratio ($\dot{q}_L = \dot{q}_M/K_G$), τ_f the friction force.

In contact mode the load velocity will be equal with the motor velocity, modified with the gear ratio, and the motion on the motor side is directly influenced by the load:

$$(CM)\begin{cases} \left(J_M + J_L/K_G^2\right)\ddot{q}_M = \tau - \tau_f \\ \dot{q}_L = \dot{q}_M/K_G \end{cases} \tag{3}$$

On the motor side, due to the backlash nonlinearity, the inertia of the mechanical system will be different in backlash mode and in contact mode.

In order to determine the condition for contact mode, denote with β the backlash gap size. For contact mode the difference between the motor shaft position and load position, modified with the gear ratio, should be equal with the backlash gap size if the machine moves in negative direction. If it moves in negative direction in contact mode, the position difference should be equal with the negative of the gap.

$$\begin{aligned} &Contact\ mode\ (CM): \\ &\quad if\ \ ((\dot{q}_M > 0)\ and\ (q_LK_G - q_M = -\beta)) \\ &\qquad or\ \ ((\dot{q}_M < 0)\ and\ (q_LK_G - q_M = \beta)) \\ &Backlash\ mode\ (BM): \\ &\quad otherwise \end{aligned} \tag{4}$$

The nonlinear model that describes the friction phenomena can be applied for controller design with difficulty in the case when the backlash is also present in the controlled mechanical system. To obtain a simpler friction model an approximate of the Stribeck model (1) will be applied.

Assume that the mechanical system moves in $(0,\ \dot{q}_{Mmax}]$ velocity domain. Consider a linear approximation for the exponential curve with two lines: d_{1_+} which crosses through the $(0, \tau_f(0))$ point and it is tangential to curve and d_{2_+} which passes through the $(\dot{q}_{Mmax}, \tau_f(\dot{q}_{Mmax}))$ point and tangential to curve (see Figure 1.) These two lines meet each other at the $\dot{q}_{Msw}$ velocity. In the domain $(0,\ \dot{q}_{Msw}]$ the d_{1_+} can be used for the linearization of the curve and d_{2_+} is used in the domain $(\dot{q}_{Msw},\ \dot{q}_{Mmax}]$. The maximum approximation error occurs at the velocity $\dot{q}_{Msw}$ for both linearizations.

The equations for d_{1_+} and d_{2_+} can be obtained using Taylor expansion:

$$\begin{aligned} d_{1_+} : F_{L1f_+}(\dot{q}_M) &= F_S + \left.\frac{\partial \tau_f(\dot{q}_M)}{\partial \dot{q}_M}\right|_{\dot{q}_M=0} \dot{q}_M = \\ &= F_S + \left(F_V - \frac{F_S - F_C}{\dot{q}_S}\right)\dot{q}_M \end{aligned} \tag{5}$$

$$\begin{aligned} d_{2_+} : F_{L2f_+}(\dot{q}_M) &= \tau_f(\dot{q}_{Mmax}) \\ &\quad + \frac{\partial \tau_f(\dot{q}_{Mmax})}{\partial \dot{q}_M}(\dot{q}_M - \dot{q}_{Mmax}) = \\ &= \tau_f(\dot{q}_{Mmax}) \\ &\quad + \left(F_V - \frac{F_S - F_C}{\dot{q}_S}e^{-\dot{q}_{Mmax}/\dot{q}_S}\right)(\dot{q}_M - \dot{q}_{Mmax}) \end{aligned} \tag{6}$$

Thus the linearization of the exponential friction model with bounded error can be described by two lines in the $(0,\ \dot{q}_{Mmax}]$ velocity domain:

$$\begin{aligned} d_{1_+} &: F_{L1f_+}(\dot{q}_M) = a_1 + b_1\dot{q}_M, \\ &\quad for\ 0\ < \dot{q}_M\ \le\ \dot{q}_{Msw} \end{aligned} \tag{7}$$

$$\begin{aligned} d_{2_+} &: F_{L2f_+}(\dot{q}_M) = a_2 + b_2\dot{q}_M, \\ &\quad for\ \dot{q}_{Msw}\ < \dot{q}_M\ \le\ \dot{q}_{Mmax} \end{aligned} \tag{8}$$

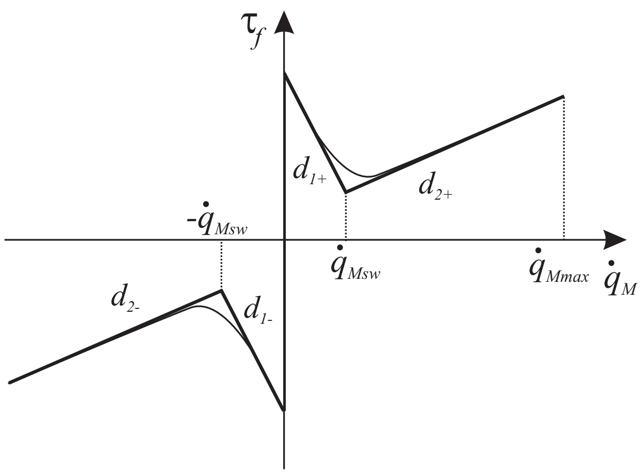

Fig. 1. Linearization of Stribeck friction model

The value of $\dot{q}_{Msw}$ can be determined from (7) and (8):

$$\dot{q}_{Msw} = \frac{a_1 - a_2}{b_2 - b_1} \tag{9}$$

Similar study can be made for negative velocities. Based on linearization, the friction can be modeled as follows:

$$F_{Lf}(\dot{q}_M) = \begin{cases} a_1 + b_1\dot{q}_M, & if\ \dot{q}_M \in (0, \dot{q}_{Msw}] \\ a_2 + b_2\dot{q}_M, & if\ \dot{q}_M \in (\dot{q}_{Msw}, \dot{q}_{Mmax}] \\ -a_1 + b_1\dot{q}_M, & if\ \dot{q}_M \in [-\dot{q}_{Msw}, 0) \\ -a_2 + b_2\dot{q}_M, & if\ \dot{q}_M \in [-\dot{q}_{Mmax}, -\dot{q}_{Msw}) \end{cases} \tag{10}$$

Note that $b_1 < 0$ and $b_2 > 0$. The friction will have different behaviors in the low velocity and in the high velocity regimes. If the absolute value of the velocity is smaller than $\dot{q}_{sw}$ the friction may induce unstable behavior and limit cycle.

This model can also effectively be used for friction identification, adaptive compensation Márton and Lantos [2007] and friction induced limit cycle prediction Márton [2008].

By approximating the friction with the linearized friction model ($\tau_f \approx \tau_{Lf}$) and considering the contact and backlash regimes (2) and (3), the mechanical system with friction and backlash can be modeled as a hybrid system. In the different partitions of the state space, different models describe the plant dynamics:

$$If\ (BM)\ and\ (|\dot{q}_M| \le \dot{q}_{Msw}): \quad \begin{cases} J_M\ddot{q}_M + b_1\dot{q}_M = \tau - a_1 sign(\dot{q}_M) \\ J_L\ddot{q}_L = 0 \end{cases} \tag{11}$$

$$If\ (BM)\ and\ (|\dot{q}_M| > \dot{q}_{Msw}): \quad \begin{cases} J_M\ddot{q}_M + b_2\dot{q}_M = \tau - a_2 sign(\dot{q}_M) \\ J_L\ddot{q}_L = 0 \end{cases} \tag{12}$$

$$If\ (CM)\ and\ (|\dot{q}_M| \le \dot{q}_{Msw}): \quad \begin{cases} \left(J_M + J_L/K_G^2\right)\ddot{q}_M + b_1\dot{q}_M = \\ \quad = \tau - a_1 sign(\dot{q}_M) \\ \dot{q}_L = \dot{q}_M/K_G \end{cases} \tag{13}$$

$$If\ (CM)\ and\ (|\dot{q}_M| > \dot{q}_{Msw}): \quad \begin{cases} \left(J_M + J_L/K_G^2\right)\ddot{q}_M + b_2\dot{q}_M \\ \quad = \tau - a_2 sign(\dot{q}_M) \\ \dot{q}_L = \dot{q}_M/K_G \end{cases} \tag{14}$$

The obtained submodels are linear, hence the theory of the hybrid linear systems can be applied to solve the control of the mechanical system in the presence of friction and backlash.

3. MOTION STABILIZATION

If the load side position and velocity ($q_L,\ \dot{q}_L$) cannot be measured, the applied controller for the stabilization of the system should rely only on measurements made on the motor side ($q_M,\ \dot{q}_M$). With the size of backlash gap unknown it is also difficult to determine whether the system is in contact mode or backlash mode. Hence for the stabilization a fixed structure linear state feedback controller is used extended with an integral term:

$$\tau = -K_P q_M - K_V \dot{q}_M - K_I \int_0^t q_M(\xi) d\xi \tag{15}$$

The controller parameters should be designed in such a way to guarantee the stability of the control system in each partition of the state space. To achieve this, the following stability theorem is applied: Let the dynamics of the hybrid system given by $\dot{\underline{x}} = f_i(\underline{x})$ with a rule for switching among the submodels. If there exists a common Lyapunov function candidate V for all submodels, which is strictly decreasing in all of the state space partitions, then the hybrid system is asymptotically stable Beldiman and Bushnell [1999].

To solve the stabilization of the system with fixed structure controller, the following strategy is applied: the parameters of the state feedback controllers are designed for one partition of the hybrid system (using the LQ design approach), then a checking relation is developed based on which the Lyapunov stability of the control system can be verified in the other partitions.

According to (11)-(14) and appropriate choice of the parameters, the state space model of the subsystem, for which the LQ controller is developed, can be described in the form given by:

$$\begin{pmatrix} \dot{q}_M \\ \ddot{q}_M \\ \dot{q}_I \end{pmatrix} = A \begin{pmatrix} q_M \\ \dot{q}_M \\ q_I \end{pmatrix} + B\tau + \begin{pmatrix} 0 \\ -sign(q_M)a/J \\ 0 \end{pmatrix} \tag{16}$$

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -b/J & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1/J \\ 0 \end{pmatrix}$$

q_I denotes the integral of the position input on the motor side: $q_I = \int_0^t q_M(\xi) d\xi$.

The standard LQ design procedure is applied to determine the controller parameters. Let Q be a positive definite symmetric matrix and r a positive scalar value. The gain vector of the controller $K = (K_P\ K_V\ K_I)^T$ is calculated as:

$$K = \frac{1}{2} r^{-1} \cdot B^T \cdot P \tag{17}$$

where the positive definite symmetric P matrix is given by the Riccati equation:

$$P \cdot A + A^T \cdot P - P \cdot B \cdot r^{-1} \cdot B^T \cdot P = -Q \tag{18}$$

The LQ controller guarantees the stability of the system in the sense of Lyapunov: it can easily be shown, that the time derivative of Lyapunov function candidate:

$$V(\underline{x}) = \underline{x}^T \cdot P \cdot \underline{x} \tag{19}$$

is always negative for $\underline{x} \neq \underline{0}$:

$$\dot{V}(\underline{x}) < -\underline{x}^T \cdot Q \cdot \underline{x} \tag{20}$$

$\underline{x}$ denotes the state vector: $\underline{x} = (q_M \ \dot{q}_M \ q_I)^T$.

Assume that the controller is determined for a given partition of the state space. It is considered that in the other partitions there is an additive uncertainty for the friction parameter: $b := b + \Delta b$ regarded to the parameter value for which the controller was designed and there is an amplification type uncertainty for the inverse of inertia: $\frac{1}{J} := \frac{\Delta(1/J)}{J}$.

If the controller is designed for the low velocity regime and the absolute value of the velocity of the machine is higher than the Stribeck velocity, the additive friction modeling error is: $\Delta b = b_2 - b_1 > 0$, otherwise $\Delta b = b_1 - b_2 < 0$.

If the controller is designed for contact mode and the backlash mode is active, the inertia modeling uncertainty is $\Delta(1/J) = \frac{J_M + J_L/K_G^2}{J_M} > 1$, otherwise $\Delta(1/J) = \frac{J_M}{J_M + J_L/K_G^2} < 1$.

Hence in the other partitions the state matrix A and the vector B can be written as the original matrices extended with modeling uncertainties:

$$A := A + \Delta A = A + \frac{(1 - \Delta(1/J)) \cdot b - \Delta(1/J) \cdot \Delta b}{J} I_2 \tag{21}$$

$$B := \Delta B \cdot B = \Delta(1/J) \cdot B \tag{22}$$

$$where \ I_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{23}$$

Consider that for a given positive Q and r the controller was determined using (18) and (17). The Lyapunov stability of the control system in the other partition is analyzed based on the Lyapunov function candidate (19). The time derivative of the Lyapunov function in the other partitions, using (18), can be written as:

$$\begin{aligned} \dot{V} &= \underline{x}^T \cdot P \cdot \dot{\underline{x}} + \dot{\underline{x}}^T \cdot P \cdot \underline{x} = \\ &= \underline{x}^T \cdot \big((A^T + \Delta A) \cdot P + P \cdot (A + \Delta A) - \\ &\quad \Delta B^2 \cdot r^{-1} \cdot P \cdot B \cdot B^T \cdot P\big) \cdot \underline{x} \\ &= \underline{x}^T \cdot \big(-Q + \Delta A \cdot P + P \cdot \Delta A - \\ &\quad (\Delta B^2 - 1) \cdot r^{-1} \cdot P \cdot B \cdot B^T \cdot P\big) \cdot \underline{x} \end{aligned} \tag{24}$$

Hence the fixed structure controller can stabilize the hybrid system in the sense of Lyapunov, if the following relation holds for all partitions of the state space:

$$-\Delta A \cdot P - P \cdot \Delta A + (\Delta B^2 - 1) \cdot r^{-1} \cdot P \cdot B \cdot B^T \cdot P + Q > 0 \tag{25}$$

With the introduced notations for parameter uncertainties it yields:

$$\begin{aligned} &-\frac{(1 - \Delta(1/J)) \cdot b - \Delta(1/J) \cdot \Delta b}{J} (I_2 \cdot P + P \cdot I_2) \\ &+ (\Delta(1/J)^2 - 1) r^{-1} \cdot P \cdot B \cdot B^T \cdot P + Q > 0 \end{aligned} \tag{26}$$

The main result of this study can be formulated as follows: If the LQ controller is designed in such way that the relation (26) hold for each submodel (11)-(14), the asymptotic stability of the control system is guaranteed.

The relation above is a sufficient condition for the stability. If the controller is designed for one of the submodels of the hybrid system, the relation (26) must be verified for all the other submodels (11)-(14).

4. SIMULATION RESULTS

Simulations were performed in $Matlab/Simulink^{TM}$ environment to demonstrate the applicability of theoretical results. The dynamics of the controlled system was simulated by implementing the models (2) and (3). For the implementation of the friction torque the original Stribeck model was used, given by (1). The parameters of the controlled mechanical system were taken as:

- Inertia of the load: $J_L = 0.5$ $[kgm^2]$
- Inertia on the motor side $J_M = 0.01$ $[kgm^2]$
- Backlash gap: $\delta = 0.001$ $[rad]$
- Gear ratio: $K_G = 5$
- Viscous friction coefficient: $F_V = 0.1$ $[Nm/rad]$
- Coulomb friction coefficient: $F_S = 0.5$ $[Nm]$
- Static Friction coefficient: $F_S = 1.5$ $[Nm]$
- Stribeck velocity: $\dot{q}_S = 0.1$ $[rad]$

The parameters of the linearized friction model (10) were determined based on relations (5) and (6), assuming that $\dot{q}_{Mmax} \to \infty$.

Two control experiments were performed. In both cases the initial motor and load positions were taken 0 and the reference position was taken $q_{Mref} = 1$ $[rad]$.

In the first experiment the LQ controller was determined for backlash mode and for high velocity regimes ($|\dot{q}_M| > \dot{q}_{Msw}$). For controller design the following matrices were applied: $Q = diag([100 \ 2.5 \ 100])$, $r = 1$. For these parameters the obtained control algorithm is:

$$\tau = 11.54\,(q_{Mref} - q_M) + 10 \int_0^t (q_{Mref} - q_M(\xi))d\xi - 1.55\dot{q}_M \tag{27}$$

With the designed controller the relation (26) was verified for the other three regions of the state space. It was found that for low velocity regimes the relation (26) does not hold. Simulation results are shown in the Figure 2. Due to the instability in the low velocity regime the motor position oscillates around the reference position, the limit cycle appears.

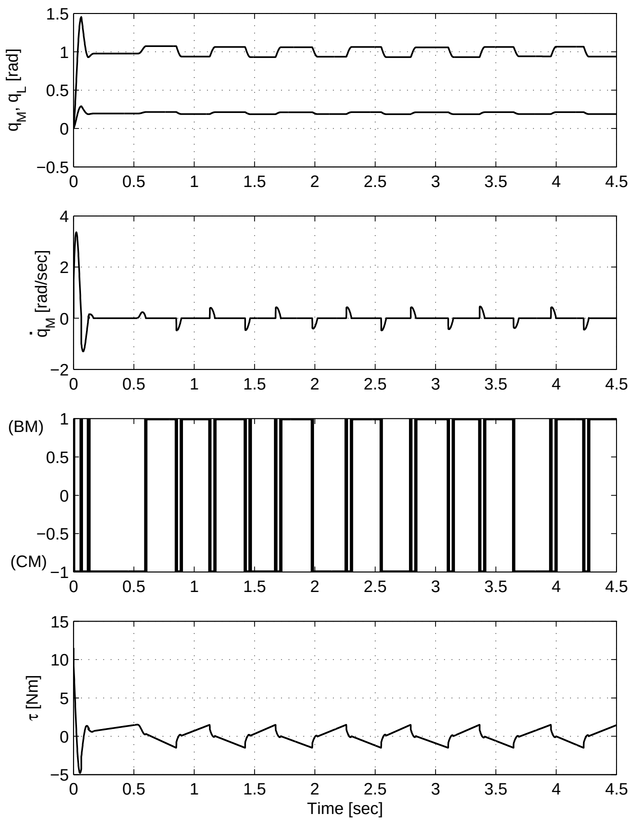

Fig. 2. Simulation results (LQ control designed for (BM) and $|\dot{q}_M| > \dot{q}_{Msw}$)

In the second experiment the LQ controller was designed with the same Q and r matrices as in the first experiment, but for contact mode and low velocity regime. The obtained control algorithm is:

$$\tau = 17.36\,(q_{Mref} - q_M) + 10 \int_0^t (q_{Mref} - q_M(\xi))d\xi$$
$$-19.98\dot{q}_M \quad (28)$$

With this controller the relation (26) holds for all regions of the controlled hybrid system. The simulation results (Figure 3) also show that the asymptotic stability of the control system is guaranteed.

5. CONCLUSIONS

In this study the control of mechanical systems with nonlinear friction and backlash was discussed with fixed structure controller when no measurements are available on the load side and with the size of the backlash gap unknown. To describe the dynamics of the controlled plant a hybrid system model was elaborated. For motion control an LQ controller was designed, based on the hybrid model. A sufficient condition for the solution of Riccati equation was formulated to guarantee the asymptotic stability of the control system in all partitions of the state space. The simulation results show that with the proposed design strategy the limit cycles that appears due to backlash and friction can be avoided.

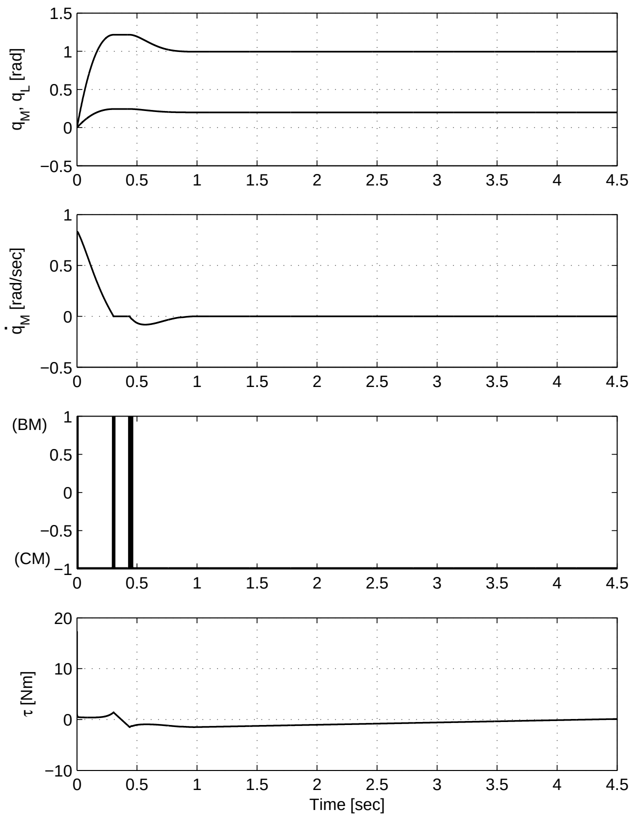

Fig. 3. Simulation results (LQ control designed for (CM) and $|\dot{q}_M| < \dot{q}_{Msw}$)

ACKNOWLEDGEMENTS

The first author research was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences. The research was also supported by the Hungarian National Research program under grant No. OTKA K 71762 and by the Control System Research Group of the Hungarian Academy of Sciences.

REFERENCES

Brian Armstrong-Hèlouvry. *Control of Machines with Friction.* Kluver Academic Press, Boston, 1991.

Antonio Barreiro and Alfonso Banos. Input-output stability of systems with backlash. *Automatica*, 42:1017–1024, 2006.

Octavian Beldiman and Linda Bushnell. Stability, linearization and control of switched systems. In *Proc. of the American Control Conference*, pages 2950–2954, San Diego, California, June 1999.

C. Caundas de Wit, H. Ollson, K. J. Åstrom, and P. Lischinsky. A new model for control of systems with friction. *IEEE Trans. on Automatic Control*, 40(3):419–425, March 1995.

Pierre Dupont, Brian Armstrong, and Friedhelm Altpeter. Single state elsatoplastic friction models. *IEEE Trans. on Automatic Control*, 47(5):787–792, May 2002.

M. Feemster, P. Vedagrbha, D. M. Dawson, and D. Haste. Adaptive control techniques for friction compensation. In *Proc. of the American Control Conference*, pages 1488–1492, Philadelphia, Pennsylvania, June 1998.

H.A. Hensen, M.J.G. van de Molengraft, and M. Steinbuch. Friction induced hunting limit cycles: A comparison between the LuGre and switch friction modell. *Automatica*, (39):2131–2137, 2003.

Adam Lagerberg and Bo Egardt. Backlash estimation with application to automotive powertrains. *IEEE Trans. Control Systems Technology*, 15(3):483–493, May 2007.

Vincent Lampaert, Jan Swevers, and Farid Al-Bender. Modification of the Leuven integrated friction model structure. *IEEE Trans. on Automatic Control*, 47(4): 683–687, April 2002.

C. Makkar, W. G. Sawyer G. Hu, and W. E. Dixon. Lyapunov-based tracking control in the presence of uncertain nonlinear parameterizable friction. *IEEE Trans. on Automatic Control*, 52(10):1988–1994, October 2007.

Lőrinc Márton. On analysis of limit cycles in positioning systems near Striebeck velocities. *Mechatronics*, 48:46–52, 2008.

Lőrinc Márton and Béla Lantos. Modeling, identification and compensation of stick-slip friction. *IEEE Trans. on Industrial Electronics*, 54(1):511–521, February 2007.

K. Menon and K. Krishnamurthy. Control of low velocity friction and gear backlash in a machine tool feed drive system. *Mechatronics*, 9:33–52, 1999.

Sepehri Nariman, Sassani F., Lawrence P.D., and Ghasempoor A. Simulation and experimental studies of gear backlash and stick-slip friction in hydraulic excavator swing motion. *ASME J. of Dynamic Systems, Measurements and Control*, 118:463–467, 1996.

Mattias Nordin and Per-Olof Gutman. Controlling mechanical systems with backlash - a survey. *Automatica*, 38:1633–1649, 2002.

D. Putra, H. Nijmeijer, and N. van deWouw. Analysis of undercompensation and overcompensation of friction in 1DOF mechanical systems. *Automatica*, 43:1387–1394, 2007.

Philipp Rostalski, Thomas Besselmann, Miroslav Baric, Femke van Belzen, and Manfred Morari. Hybrid approach to modeling, control and state estimation of mechanical systems with backlash. *International Journal of Control*, 80(11):1729–1740, November 2007.

S. Suraneni, I.N. Kar, O.V. Ramana Murthy, and R.K.P. Bhatt. Adaptive stick slip friction and backlash compensation using dynamic fuzzy logic system. *Applied Soft Computing*, 6:26–37, 2005.

Jan Swevers, Farid Al-Bender, Chris G. Ganseman, and Tutuko Prajogo. An integrated friction model structure with improved presliding behavior for accurate friction compensation. *IEEE Trans. on Automatic Control*, 45 (5):675–685, April 2000.

Gang Tao and Petar Kokotovic. *Adaptive Control of Systems with Sensor and Actuator Nonlinearities.* John Wiley & Sons, New York, 1996.

S. Tarbouriech and C. Prieur. Stability analysis for sandwich systems with backlash: an LMI approach. In *Proc. of 5th IFAC Symposium on Robust Control Design*, Toulouse, France, July 2006.

Wen-Fang Xie. Sliding-mode-observer-based adaptive control for servo actuator with friction. *IEEE Trans. on Industrial Electronics*, 54(3):1517–1528, June 2007.