

# 'FLAT PHASE' PID CONTROLLERS

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Abstract: Flat Phase PID Controllers have the property that the phase of the transfer function round the associated feedback loop is constant or flat around the design frequency, with the aim that the phase margin and overshoot to a step response is unaffected when the gain of the device under control changes. Such designs have been achieved using Bode Integrals and by ensuring the phase is the same at two frequencies. This paper extends the 'two frequency' controller and describes a novel three frequency controller. The different design strategies are compared. *Copyright © 2008 R.J.Mitchell*

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## 1. INTRODUCTION

The PID controller is the most common practical solution to control problems (Aström and Hagglund, 2006), and there are numerous ways of determining its parameters. Recently papers have described how the controller parameters can be found using Bode's ideal loop transfer function (Bode, 1948). Here, at least at key frequencies, the loop transfer function,  $L(j\omega)$ , that round the feedback loop, has the form

$$L(j\omega) = \left( \frac{\omega_c}{j\omega} \right)^r \quad (1)$$

Here  $\omega_c$  is the gain crossover frequency, where  $|L(j\omega)|$  is unity, and  $r$  is the slope of the  $\log(\text{gain})$  versus  $\log(\omega)$  graph. Defining  $y$  as  $-PM/\pi$ , where  $PM$  is the phase margin in radians as specified by the designer, then  $r$  is  $-2(1-y)$ , which is typically a fraction between -1 and -2. As the associated phase is  $\pi/2$  multiplied by that slope, the phase is  $-\pi + PM$ , so at  $\omega_c$  the phase margin is achieved.

The associated Nyquist diagram is a straight line, through the origin, at an angle of  $PM$  below the negative real axis.

The designs based on this concept include those by Barbosa et al (2005), the 'flat phase' controller of Chen and Moore (2005) and the 'two point' flat

phase controller of Mitchell (2006c). The controller of the first paper is tuned by minimising the integral square error between the closed loop step responses of the controlled system and the system defined by equation (1). The two other methods seek to ensure that the loop transfer function is of that form at key design frequencies. This paper extends these methods and so the discussion will concentrate on them.

The significant property of these controllers is that, as the phase is constant around the design frequency, were the gain of the plant under control to change, the phase would not, and hence the phase margin would remain unaltered. In consequence, the overshoot of the system step response will be constant, the system having an 'iso-damping' property, being robust to changes in plant gain.

The PID controller of Chen and Moore (2005) is tuned so that at a specified frequency the loop transfer function has gain  $\cos(PM)$  and phase  $-\pi+PM$  and the phase is locally constant or flat. Hence, around the given frequency, the associated Nyquist locus follows the straight line mentioned earlier.

Mitchell (2006b) demonstrates how the associated equations can be greatly simplified and considers whether the gain should be unity or  $\cos(PM)$  at the desired frequency, as well as comparing the performance of the PID controller with that of

systems designed using Bode's maximum available feedback method (Bode, 1948) and Mitchell (2006a) which is also in effect a flat phase system.

Mitchell (2006c) considers an alternative flat phase PID controller, where the aim is that the Nyquist locus passes through two points on the same straight line. The designer specifies two frequencies and the controller parameters are set so that at those two frequencies the phase is  $-\pi+PM$ , and the gain is that specified at one of those two frequencies.

The initial aim of the two point controller was to address an issue raised by Chen and Moore (2005), namely that although the phase is flat at the design frequency the 'width' of the flat phase region is not known. Specifying the performance at two frequencies only partially addresses this – what would be better is to in effect specify the range of gains within which the plant gain can change. This is addressed in this paper.

Another contribution in this paper is a three point flat phase controller. This not only sets the overall gain and the integral and derivative terms, but also the time constant of the filter of the derivative term, which is used to reduce the effects of high frequency measurement noise (Aström and Hagglund, 2006).

Finally, so as to compare this three point controller fairly with the flat phase and two point PID controllers, such a filter is added to them and the effect of that investigated.

The organisation of the paper is as follows. Some relevant details are given of the two point controller, and this is followed by the refinement to allow its flat phase range to be specified. The novel three point controller is then described. Results of experiments using these controllers are presented. This is followed by conclusions and areas of further work.

## 2. PID CONTROLLERS

In this paper, the plant under control is denoted by  $P(s)$  and the PID controllers used are of the form

$$C(s) = K_p \left( 1 + \frac{1}{sT_i} + sT_d \right) \quad (2)$$

Or, if the derivative term is to be filtered,

$$C(s) = K_p \left( 1 + \frac{1}{sT_i} + \frac{sT_d}{1+sT_d/N} \right) \quad (3)$$

For the two point controller, the designer specifies two frequencies and that the loop transfer function should have a certain gain and phase at the high frequency end of the 'flat phase' region, and the same phase at the low frequency end. Suppose these two frequencies are given by  $\omega_1$  and  $\omega_2$  and the phase at these frequencies is to be  $-\pi + \Phi$ , then

$$\angle (C(\omega) * P(\omega)) = -\pi + \Phi \text{ at } \omega_1 \text{ and } \omega_2 \quad (4)$$

As

$$\angle C(\omega) = \tan^{-1} \left( \omega T_d - \frac{1}{\omega T_i} \right) \quad (5)$$

It follows that

$$\omega T_d - \frac{1}{\omega T_i} = \tan(\Phi - \pi - \angle P(j\omega)) = t_p(\omega) \quad (6)$$

Knowing  $\omega_1$  and  $\omega_2$ , it is straightforward to calculate  $t_p(\omega_1)$  and  $t_p(\omega_2)$ . Mitchell (2006c) shows that

$$T_i = \frac{\omega_1^2 - \omega_2^2}{\omega_1 \omega_2 (\omega_2 t_p(\omega_1) - \omega_1 t_p(\omega_2))} \quad (7)$$

And

$$T_d = (t_p(\omega_1) + \frac{1}{\omega_1 T_i}) / \omega_1 \quad (8)$$

Also, as

$$\begin{aligned} |C(j\omega_1)| &= K_p \sqrt{1 + \left( \omega_1 T_d - \frac{1}{\omega_1 T_i} \right)^2} \\ &= K_p \sqrt{1 + \tan^2(\angle C(j\omega_1))} \\ &= K_p \sec(t_p(\omega_1)) \end{aligned} \quad (9)$$

Then, if the loop gain is to be  $G$  at  $\omega_1$ , then

$$K_p = \frac{|\cos(t_p(\omega_1))| G}{|P(j\omega_1)|} \quad (10)$$

One problem with the above is that the user must specify the two angular frequencies where the loop phase is to be the desired value. Given that the system is aimed to be iso-robust to changes in plant gain, it is more intuitive to specify the frequency for the middle of the flat phase region, and to then specify the gain at each end of that region, from which the key frequencies are then calculated. As the phase at any frequency is  $\pi/2$  times the slope of the log gain versus log frequency graph at that frequency, and for this controller that slope is  $r$ , from (1), which equals  $-2(1-y)$ , then the relationship between the gains at two frequencies  $\omega_1$  and  $\omega_2$  can be estimated by

$$\frac{\text{Gain at } \omega_2}{\text{Gain at } \omega_1} = \left( \frac{\omega_2}{\omega_1} \right)^{2(1-y)} \quad (11)$$

Hence, if the controller is designed so that the system has a given gain  $G$  at  $\omega_0$  and the system is to be iso-robust if the gain is increased/decreased by a factor  $gfac$ , then  $\omega_2$  and  $\omega_1$  are determined by

$$\omega_2 = \omega_0 * gfac^{1/2(1-y)} \quad (12)$$

And

$$\omega_1 = \omega_0 / gfac^{1/2(1-y)} \quad (13)$$

Key to the design is the choice of  $\omega_c$  which Chen and Moore note depends on the system dynamics. If  $G$  is unity, then  $\omega_c$  is the crossover frequency which relates to the step response time to peak, the design requirement for which can be used to select  $\omega_c$ .

### 3. THREE POINT CONTROLLER

Mitchell (2006c) reports that the two point controller has the correct phase at two frequencies, but usually between these frequencies the phase is different. Hence around the main frequency, it is not as flat as the systems designed by Chen and Moore (2005). However, Mitchell (2006c) reports that the two point controller is more robust as regards overshoot to changes in the gain of the plant under control.

In consequence, it was decided to investigate a three point controller, where the system will have the desired phase lag at three specified frequencies. For this to work, the design information needs to be able to determine  $T_d$ ,  $T_i$  and  $N$  as well as  $K_p$ , in equation 3. In fact, to simplify the algebra, that equation is rearranged into the following

$$C(s) = K \frac{s^2 + as + b}{s(1 + sc)} \quad (14)$$

The phase of this is as follows

$$\angle C(j\omega) = \tan^{-1} \left( \frac{a\omega}{b - \omega^2} \right) - \tan^{-1}(\omega c) - \frac{\pi}{2} \quad (15)$$

It is required that, at the designed frequencies, this must equal  $-\pi + \Phi - \angle P(j\omega)$ , hence

$$\tan \angle C(j\omega) = \frac{\frac{a\omega}{b - \omega^2} - \omega c}{1 + \frac{a\omega}{b - \omega^2} \omega c} = \tan \left( -\frac{\pi}{2} + \Phi - \angle P(j\omega) \right)$$

Or

$$\frac{a\omega - bc\omega + c\omega^3}{b - \omega^2 + ac\omega^2} = \cot(\Phi - \angle P(j\omega)) = v \quad (16)$$

Note, the definition of the variable  $v$ . Hence

$$c\omega^3 + (1 - ac)v\omega^2 + (a - bc)\omega - vb = 0 \quad (17)$$

Suppose the three design frequencies are stored in a column vector,  $\omega$ , and let there be the following three vectors  $\mathbf{x} = \omega^3$ ;  $\mathbf{y} = v\omega^2$ ;  $\mathbf{z} = \omega$ ; then

$$c\mathbf{x} + (1 - ac)\mathbf{y} + (a - bc)\mathbf{z} - b\mathbf{v} = 0 \quad (18)$$

This can be expressed in matrix form

$$\begin{bmatrix} y_1 & z_1 & x_1 & v_1 \\ y_2 & z_2 & x_2 & v_2 \\ y_3 & z_3 & x_3 & v_3 \end{bmatrix} \begin{bmatrix} 1 - ac \\ a - bc \\ c \\ -b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (19)$$

Suppose  $\mathbf{A}$  is the four by three matrix in (19). Using row-echelon techniques,  $\mathbf{A}$  can be reduced to being triangular and then  $\mathbf{A}[2,1]$  can be set to 0, so:

$$\begin{bmatrix} y_1 & 0 & x_1 & v_1 \\ 0 & z_2 & x_2 & v_2 \\ 0 & 0 & x_3 & v_3 \end{bmatrix} \begin{bmatrix} 1 - ac \\ a - bc \\ c \\ -b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

Once  $\mathbf{A}$  is in this form, the rows in the matrix equation can be processed as follows, where some intermediary variables  $p$ ,  $q$  and  $r$  are determined.

$$\text{From row 3 } b = \frac{x_3}{v_3} c = p c;$$

$$\text{From row 2 } a = bc + \frac{(v_2 p - x_2)}{z_2} c = pc^2 + qc$$

$$\text{From row 1 } (1 - ac) + \frac{x_1 - v_1 p}{y_1} c = 1 - ac + r c = 0$$

These can be rearranged to form the cubic equation:

$$-pc^3 - qc^2 + rc + 1 = 0 \quad (21)$$

Solving this polynomial gives  $c$ , from which  $a$  and  $b$  are easily found.

Alternatively, by equating (13) and (3) one can establish the following relationships between  $K$ ,  $a$ ,  $b$  and  $c$  and  $K_p$ ,  $T_i$ ,  $T_d$  and  $N$ , and hence it is possible to find these values using  $c$ ,  $p$  and  $q$ .

$$T_i = a/b - c = q/p \quad (22)$$

$$T_d = 1 / (a - bc) - c = 1/qc - c \quad (23)$$

$$N = T_d / c \quad (24)$$

$$K_p = K * (a - bc) = K * qc \quad (25)$$

This can be implemented easily in MATLAB.

### 4. FILTERING THE DERIVATIVE TERM

The three point controller automatically generates a value for  $N$ , which is used in determining the time constant associated with the filter of the derivative term in the controller (whose aim is to reduce the effects of measurement noise in the system).

It seems appropriate, when comparing this controller with the flat phase and two point controllers, to incorporate such a filter. This can be achieved easily by using the values for  $K_p$ ,  $T_i$  and  $T_d$  calculated by the design method and adding a filter.

One drawback here is that the flat phase and two point controllers are designed assuming that the controller used is that shown in equation (2). Including the derivative filter means that the actual performance may not be the same as what is desired.

Modifying the design methods to include N is however non trivial, and is left for further work.

Values for N typically used are between 2 and 20, Aström and Hagglund, (2006) In the experiments reported here a constant value of 10 was used. As this can adjust the performance at key frequencies, a simple compensation employed was to adjust slightly the specified phase margin to compensate for a phase difference due to the filter. This was done empirically with the aim of achieving the same overshoot as that obtained using the equivalent controller without the filter.

## 5. RESULTS

The different controllers are here tested on two plants used in the literature.

The first plant considered in Chen and Moore (2005) is the following, having repeated poles (as occur in similar examples in Aström and Hagglund, (2006).

$$P_1(s) = \frac{1}{(1+s)^5} \quad (26)$$

The flat phase controller for a specified phase of  $45^\circ$  at  $\omega = 0.4$  rad/s is (Mitchell 2006b)

$$C_{FP1}(s) = 0.9211 \left( 1 + \frac{1}{s1.9608} + s1.9686 \right) \quad (27)$$

With the filter, a similar overshoot was obtained by specifying the phase as  $43.8^\circ$ . The controller is

$$C_{FPN1}(s) = 0.9303 \left( 1 + \frac{1}{s1.9093} + \frac{s1.9901}{1+s0.199} \right) \quad (28)$$

A two point controller was then designed for the same phase and  $\omega$  where the two points specified were where the gain was 0.1 away from that at  $\omega$ .

$$C_{2P1}(s) = 0.9225 \left( 1 + \frac{1}{s1.9020} + s2.0584 \right) \quad (29)$$

With the filter, the specified phase was also changed to  $43.8^\circ$ . The controller is

$$C_{2PN1}(s) = 0.9149 \left( 1 + \frac{1}{s1.8516} + \frac{s2.0814}{1+s0.2081} \right) \quad (30)$$

The three point controller was designed to pass through the same two points as the two point controller, and also to provide a phase of  $45^\circ$  at 0.4 rad/s. The controller is

$$C_{3PA1}(s) = 0.755 \left( 1 + \frac{1}{s1.6069} + \frac{s2.5284}{1+s0.5871} \right) \quad (31)$$

A second three point controller was then designed where the three points were further apart.

$$C_{3PB1}(s) = 0.7637 \left( 1 + \frac{1}{s1.6164} + \frac{s2.5069}{1+s0.5519} \right) \quad (32)$$

The ‘flatness’ of the phase of the loop transfer function for each controller was measured, and its variation from the specified  $135^\circ$  is shown in table 1.

**Table 1** For the given angular frequencies, difference between actual phase of the loop and that desired.

$\omega$	$C_{FP1}$	$C_{FPN1}$	$C_{2P1}$	$C_{2PN1}$	$C_{3PA1}$	$C_{3PB1}$
0.35	1.02	1.09	0.535	0.689	0.206	0.16
0.36	0.71	0.782	0.27	0.44	0.099	0.056
0.37	0.45	0.53	0.08	0.26	0.035	-0.002
0.38	0.25	0.31	-0.05	0.08	0.004	-0.024
0.39	0.10	0.14	-0.13	-0.02	-0.005	-0.020
0.4	0	-0.00	-0.16	-0.1	0	0
0.41	-0.06	-0.12	-0.14	-0.14	0.007	0.026
0.42	-0.09	-0.21	-0.08	-0.16	0.006	0.046
0.43	-0.09	-0.3	0.01	-0.17	-0.02	0.052
0.44	-0.07	-0.38	0.12	-0.17	-0.06	0.034
0.45	-0.04	-0.45	0.26	-0.17	-0.14	-0.016

Over this range of frequencies the phase of the systems with the three point controllers varies less than the flat phase controller, which in turn varies less than the systems using the two point controller.

Table 2 shows what happens when the plant gain changes: that is when  $P(s)$  becomes  $P(s) * \text{Gain}$ . For the different controllers, the time-to-peak,  $T_{pk}$ , percentage overshoot, %os and settling time to within 2% of the steady state value  $T_{set}$ , are shown.

**Table 2** Variation of time to peak, overshoot and settling time when gain of  $P(s)$  changes.

$C(s)$	Gain	$T_{pk}$	%os	$T_{set}$
$C_{FP1}$	0.8	11.043	16.5	26.81
	1.0	9.841	17.6	31.12
	1.2	8.234	17.2	29.25
$C_{FPN1}$	0.8	10.943	16.0	26.41
	1.0	9.356	15.8	30.25
	1.2	6.165	21.2	28.42
$C_{2P1}$	0.8	10.986	17.4	33.13
	1.0	9.836	18.3	31.59
	1.2	8.754	17.4	29.62
$C_{2PN1}$	0.8	11.008	16.7	26.62
	1.0	9.801	16.2	31.02
	1.2	6.096	20.9	28.99
$C_{3PA1}$	0.8	11.966	16.4	34.73
	1.0	7.199	15.8	33.03
	1.2	6.235	25.4	30.96
$C_{3PB1}$	0.8	11.908	16.6	34.91
	1.0	7.216	15.4	33.00
	1.2	6.200	24.8	30.89

This shows the two point controllers to be the most iso-robust to changes of plant gain, as regards keeping overshoot and time to peak constant. Settling times are long – this seems to be characteristic of these controllers: further work is needed here.

Figure 1 shows the step response for all six controllers; in the same order as in table 2.

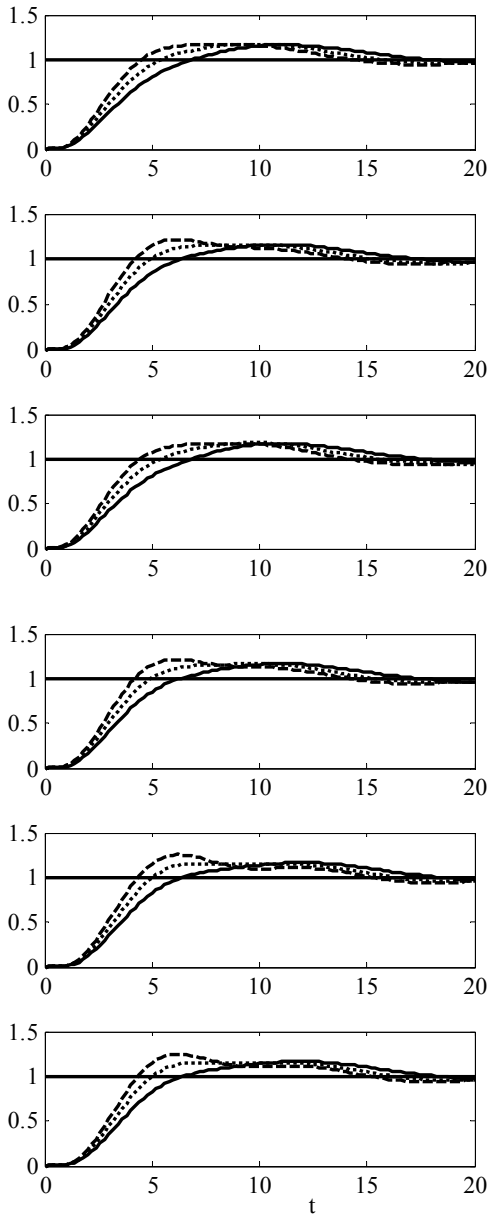


Fig 1. Variation in step responses for the systems with the six controllers,  $C_{FP1}$   $C_{FPN1}$   $C_{2P1}$   $C_{2PN1}$   $C_{3PA1}$   $C_{3PB1}$  respectively – solid lines are gain factor = 0.8, dots are gain factor = 1, dashes are gain factor = 1.2

The second plant tested is that found in Aström and Hagglund, (2006) with four distributed lags.

$$P_2(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)} \quad (33)$$

The flat phase controller for  $\omega_t = 16$  rad/s,  $\Phi_t = 48^\circ$  is

$$C_{FP2}(s) = 18.9495 \left( 1 + \frac{1}{s0.6514} + s0.0318 \right) \quad (34)$$

Here, when the derivative term is filtered, the controller is

$$C_{FPN2}(s) = 18.9495 \left( 1 + \frac{1}{s0.6514} + \frac{s0.0318}{1+s0.0032} \right) \quad (35)$$

A two point controller was then designed for the same phase and  $\omega = 15.7$  rad/s where the two points specified were where the gain was 0.1 away from that at  $\omega$ .

$$C_{2P2}(s) = 20.1672 \left( 1 + \frac{1}{s0.3216} + s0.0379 \right) \quad (36)$$

With the filter, the controller is

$$C_{2PN2}(s) = 20.1672 \left( 1 + \frac{1}{s0.3216} + \frac{s0.0379}{1+s0.0038} \right) \quad (37)$$

The three point controller passing through the same two points as that for  $C_{2P2}$ , and the midpoint is

$$C_{3PA2}(s) = 30.0191 \left( 1 + \frac{1}{s0.3178} + \frac{s0.0382}{1+s0.0006} \right) \quad (38)$$

The three point controller where the end points are defined by a gain of 0.2 away from the centre is

$$C_{3PB2}(s) = 30.2087 \left( 1 + \frac{1}{s0.3205} + \frac{s0.0380}{1+s0.0001} \right) \quad (39)$$

The Nyquist locus for  $C_{FP2}$  is the dotted line in Fig 2, whereas that for  $C_{2P2}$  follows the  $48^\circ$  phase margin line in the figure. For  $C_{FP2}$  the phase flat is where the gain is  $\cos(48)$  rather than unity, the phase exceeds  $48^\circ$  when the gain is unity. Hence the flat phase controller was redesigned so that the gain is unity at the desired frequency. The associated Nyquist locus is the dashed line in Fig 2.

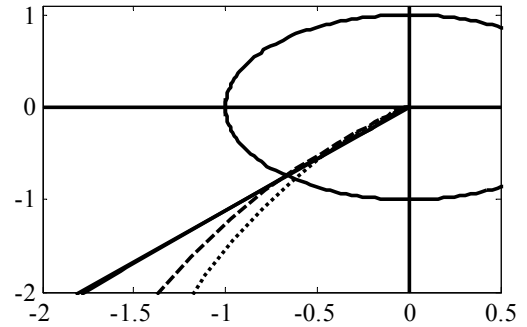


Fig 2. Nyquist Diagrams for controllers of  $P_2(s)$ : dotted line for  $C_{FP1}$ , dashed line for  $C_{FPB2}$ , solid line for  $C_{2P2}$ ; unit circle also shown

The revised controllers without and with the derivative filter are as follows.

$$C_{FPB2}(s) = 28.3195 \left( 1 + \frac{1}{s0.6514} + s0.0318 \right) \quad (40)$$

and

$$C_{FPNB2}(s) = 28.3195 \left( 1 + \frac{1}{s0.6514} + \frac{s0.0318}{1+s0.0032} \right) \quad (41)$$

The Nyquist locus for all the two point and three point controllers follow the  $48^\circ$  phase margin line very closely in the region shown, whereas those for the other flat phase controllers do not. Hence the phase of the two and three point controllers vary much less than the controllers designed using Chen and Moore's approach.

Table 3 shows what happens when the plant gain changes ( $P(s)$  being multiplied by factors 0.8, 1.0, and 1.2). Again the variation in the time to peak, overshoot and settling times of the associated step responses are shown.

Table 3 Variation of time to peak, overshoot and settling time when gain of  $P_2(s)$  changes.

C(s)	Gain	$T_{pk}$	%os	$T_{set}$
$C_{FP2}$	0.8	0.280	18.1	1.02
	1.0	0.241	19.9	0.89
	1.2	0.213	21.4	0.79
$C_{FPN2}$	0.8	0.278	18.1	1.01
	1.0	0.238	20.0	0.88
	1.2	0.209	21.6	0.79
$C_{2P2}$	0.8	0.273	26.5	0.68
	1.0	0.235	26.5	0.80
	1.2	0.207	26.5	0.76
$C_{2PN2}$	0.8	0.270	26.4	0.54
	1.0	0.231	26.6	0.46
	1.2	0.201	26.7	0.74
$C_{3PA2}$	0.8	0.208	26.5	0.75
	1.0	0.178	26.4	0.67
	1.2	0.156	26.3	0.61
$C_{3PB2}$	0.8	0.208	26.5	0.76
	1.0	0.178	26.4	0.67
	1.2	0.156	26.3	0.61
$C_{FPB2}$	0.8	0.213	21.3	0.80
	1.0	0.184	23.0	0.69
	1.2	0.163	24.2	0.62
$C_{FPNB2}$	0.8	0.209	21.6	0.79
	1.0	0.180	23.4	0.68
	1.2	0.160	24.9	0.61

An overshoot of 26% for the two and three point controllers whose Nyquist loci follow the straight line is consistent with the overshoot expected for a system whose loop transfer function is as shown in (1) with phase margin  $48^\circ$ , according to Barbosa et al (2004).

Clearly the two and three point controllers are far more iso-robust than the other flat phase controllers. In fact, for this plant, the gain can be changed by a much larger amount for these controllers, as is shown in table 4.

#### CONCLUSIONS AND FURTHER WORK

A novel three point flat phase controller has been developed, as well as extensions to a two point flat phase controller. These have been tested on two very different plants and compared with a flat phase controller designed using Bode Integrals.

Table 4 Variation of time to peak, overshoot and settling time for larger change of gain of  $P_2(s)$

C(s)	Gain	$T_{pk}$	%os	$T_{set}$
$C_{2P2}$	0.25	0.604	25.9	1.80
	0.5	0.375	26.2	1.01
	1.0	0.231	26.6	0.46
	1.5	0.172	27.0	0.66
	2	0.138	27.5	0.55
	3	0.100	29.5	0.41
	4	0.080	32.6	0.32
$C_{3PB2}$	0.25	0.458	26.3	1.32
	0.5	0.285	26.5	0.72
	1.0	0.178	26.4	0.67
	1.5	0.133	26.3	0.53
	2	0.107	26.4	0.43
	3	0.078	27.4	0.17
	4	0.063	29.4	0.23

For the plant with repeated poles, the two point controller is marginally the best; for the plant with distributed poles, the three point controller is marginally better than the two point controller. The controllers need to be tested on more plants for further conclusions to be drawn, but the approaches seem sound. Further work is also needed to see if it is possible to incorporate the filter of the derivative term in the design, and to investigate the longer than expected settling times.

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