A METHOD FOR FINDING GOOD VALUES OF ADAPTATION GAINS

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Abstract: Choosing the adaptation gains for a model reference adaptive controller is a complex matter. Low values of the adaptation gains may result in very slow adaptation, while high adaptation gains may lead to instability in the adaptation process. Good values of them depend on many factors in this type of non-linear system; the input amplitude and frequency, the process performance specification and the form of the controller, being the most important.
This paper describes the search for good values of the adaptation gains using the MIT Rule based adaptive controller as an example. The simulations are performed using LABVIEW. The method developed is demonstrated in the real- time, speed control of a DC motor.

Keywords: MIT Rule, adaptation gain, simulation, real-time control.

1. THE MIT RULE (Osborn, *et al.,* 1961; Astrom and Wittenmark, 1995)

The MIT Rule outlined in Figure 1, is a gradient based controller

$$\frac{dP}{dt} = -\gamma \frac{\partial e^2(t)}{\partial P} = -\alpha e(t)\frac{\partial e(t)}{\partial P}$$

where P are the controller parameters and α are the adaptation gains.

$$e(t) = Y_p(t) - Y_m(t)$$

This is a particular case of state feedback control (McQuade and Rurua, 2000).

For this controller

$$\frac{\partial e(t)}{\partial P_0} \approx r(t) \text{ and } \frac{\partial e(t)}{\partial P_1} \approx Y_p(t)$$

A typical response of the controller for

$$G_m(S) = \frac{3}{S+1} \text{ and } G_p(S) = \frac{1}{S}$$

is given in Figure 2 for $\alpha_0 = -5 \quad \alpha_1 = 5$

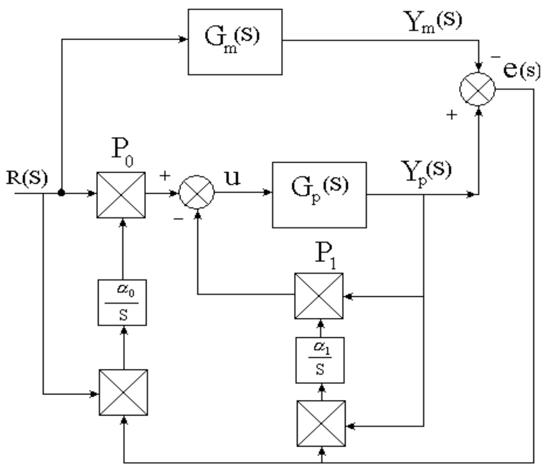

Fig. 1. MIT Rule for first order systems

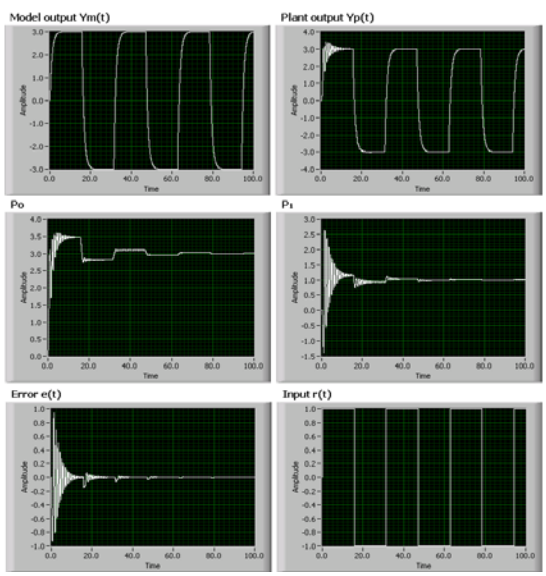

Fig 2. Controller performance for $\alpha_0 = -5$ and $\alpha_1 = 5$

2. STABILITY

A first step in the search for a good value of α is to find the range of values for which the system is stable. There are many techniques available for doing this (Rurua, 2004) but in the context of this paper we use Lyapunov's second method (Cook, 1986).

Please note that for clarity of presentation the operators (S) and (t) are removed.

Assume the plant and the model transfer functions are

$$G_p = \frac{B}{S+C} \text{ and } G_m = \frac{A}{S+A}$$

where B and C are constants and are of unknown value. Referring to Figure 1, u is the control input giving

$$u = P_0 r - P_1 Y_p \tag{1}$$

where P_0 and P_1 are to be chose so that the closed-loop transfer function, from the set-point signal r to the plant output Y_p equals the transfer function of the model.

The values of P_0 and P_1 needed so that the plant output Y_p equals Y_m are derived from

$$\frac{Y_p}{r} = \frac{P_0 B}{S + C + P_1 B} = \frac{A}{S+A}$$

giving the required values

$$\hat{P}_0 = \frac{A}{B} \text{ and } \hat{P}_1 = \frac{A-C}{B} \tag{2}$$

where B and C are unknown, or subject to change. The adaptive algorithm is used to adjust the controller gains P_0 and P_1 so that they asymptotically approach the required values $\hat{P}_0$ and $\hat{P}_1$.

The reference model G_m generates an output Y_m in response to the reference input r, so that

$$\dot{Y}_m = A(r - Y_m)$$

and for the closed loop plant

$$\dot{Y}_p = P_0 B r - (C + P_1 B) Y_p$$

The error signal is given by

$$e = Y_p - Y_m$$

and

$$\dot{e} = P_0 B r - (C + P_1 B) Y_p - A(r - Y_m)$$

Substituting for Y_m using $Y_m = Y_p - e$ leads to

$$\dot{e} = B(P_0 - \hat{P}_0) r - B(P_1 - \hat{P}_1) Y_p - Ae$$

So that

$$\dot{e} = B r \theta_0 - B Y_p \theta_1 - Ae$$

where $\theta_0 = P_0 - \hat{P}_0$ and $\theta_1 = P_1 - \hat{P}_1$ (3)

By selecting a possible Lyapunov function

$$V = e^2 + \lambda_0 \theta_0^2 + \lambda_1 \theta_1^2$$

where λ_0 and λ_1 are arbitrary constants, gives

$$\dot{V} = 2e\dot{e} + 2\lambda_0 \theta_0 \dot{\theta}_0 + 2\lambda_1 \theta_1 \dot{\theta}_1 \tag{4}$$

Using the adaptation scheme

$$\dot{P}_0 = \alpha_0 r e = \dot{\theta}_0 \text{ and } \dot{P}_1 = \alpha_1 Y_p e = \dot{\theta}_1$$

where α_0 and α_1 are the adaptation gains.

Thus

$$\dot{V} = 2e(Br\theta_0 - BY_p\theta_1 - Ae) + 2\lambda_0\theta_0\alpha_0 re + + 2\lambda_1\theta_1\alpha_1 Y_p e \tag{5}$$

For $\lambda_0 = \dfrac{B}{\alpha_0}$, $2eBr\theta_0 = 2\lambda_0\theta_0\alpha_0 re$ for $\alpha_0 < 0$

and for $\lambda_1 = \dfrac{B}{\alpha_1}$, $2eBY_p\theta_1 = 2\lambda_1\theta_1\alpha_1 Y_p e$ for $\alpha_0 > 0$

So

$$\dot{V} = -2Ae^2 \tag{6}$$

Thus the system is stable for A and B positive. This implies that there is no upper bound on the magnitude of the adaptation gains. Thus for stability $\alpha_0 < 0$ $\alpha_1 > 0$.

Other factors affecting stability, such as unmodelled dynamics, are studied elsewhere (Ashraf and McQuade, 1993; Ashraf, 1993).

3. GOOD VALUES OF α

In what follows it is assumed that the magnitudes of the two adaptation gains are equal. The response of the controller for a relatively large value of α is given in Figure 2. Figure 3 shows the response for α relatively small. In both cases it takes some time for the controller parameters (P) to settle to their nominal values $(\hat{P})$.

As shown in Figure 2 the controller adjusts the parameters P_0 and P_1 so that $Y_p(t) = Y_m(t)$ for a given input $r(t)$. In this sense $Y_m(t)$ is the performance specification for the controlled system. Note that P_0 converges to 3 and P_1 converges to 1, for this case.

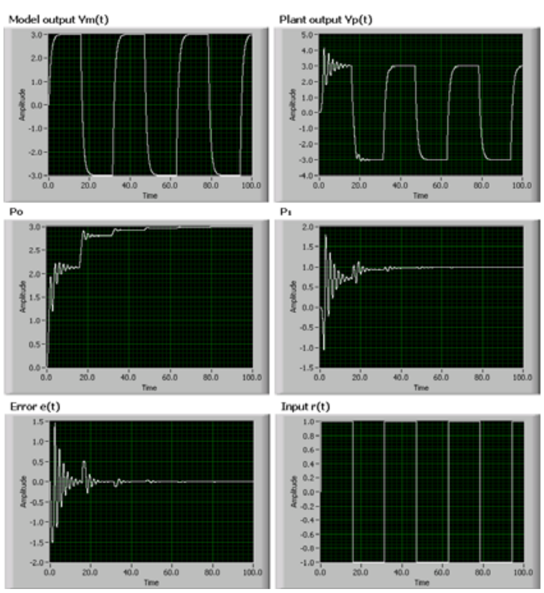

Fig. 3. Controller performance for $\alpha_0 = -1$ and $\alpha_1 = 1$

However in Figure 4, for the case of $\alpha = 3$ the system converges to the nominal parameter $(\hat{P})$ values very quickly. We define this value of $\alpha = \alpha_B = 3$ as being a good value for this case. Our aim then is to have the controller parameters converge to their nominal values in minimum time.

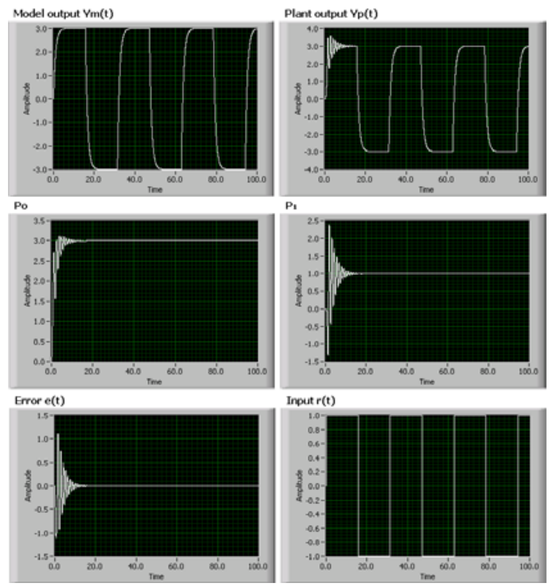

Fig. 4. Controller performance for $\alpha_0 = -3$ and $\alpha_1 = 3$

4. SEARCH FOR GOOD VALUES

An issue in this search is how to find the nominal parameter values $(\hat{P})$. To do this the controller may be used as an identifier, where the value of α is not critical. In the absence of an objective function related directly to convergence time then resort was made to more traditional optimisation techniques based on the performance criteria ISE and ITSE (Stout, 1950), where

$$\min_{\alpha} ISE = \min_{\alpha} \int_0^T e^2(t)dt \tag{7}$$

$$\min_{\alpha} ITSE = \min_{\alpha} \int_0^T te^2(t)dt \tag{8}$$

As shown in Figure 5 these two objective functions predicted best α values much bigger then that defined as good earlier. So the objective functions

$$\min_{\alpha} I\alpha = \min_{\alpha} \int_0^T \left[\left(P_0 - \hat{P}_0\right)^2 + \left(P_1 - \hat{P}_1\right)^2 \right] dt$$

and

$$\min_{\alpha} I\alpha T = \min_{\alpha} \int_0^T t\left[\left(P_0 - \hat{P}_0\right)^2 + \left(P_1 - \hat{P}_1\right)^2\right]dt$$

were used. These two arose naturally from the form of the Lyapunov Function given above, with $I\alpha T$ having a similarity to ITSE.

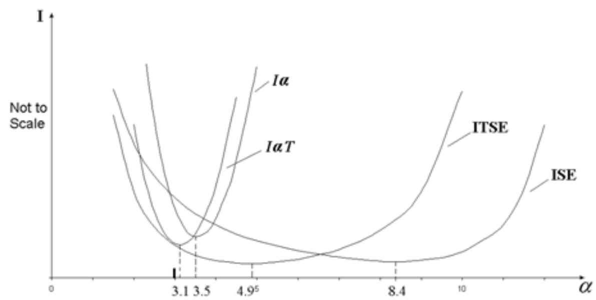

Fig. 5. Performance criteria

Figure 5 shows that $\min_{\alpha} I\alpha T$ is a good predictor of good α s. This was verified in many simulation examples. For this case $\min_{\alpha} I\alpha T$ indicates that $\alpha = 3.1$ is the best value, whereas the experimentally determined value was $\alpha_B = 3$.

5. APPLYING THE METHOD

The conclusion from this exploration is that $\min_{\alpha} I\alpha T$ predicts close to good α 's for a range of examples. Thus, although we have failed to formulate a theoretical approach to find the optimum α , we do have a way of finding good α 's. An approach to tuning this controller is given in Figure 6.

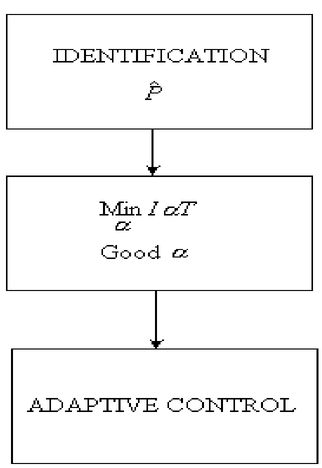

Fig. 6. An approach to tuning the MIT Rule

6. SPEED CONTROL OF A DC MOTOR

The method is applied to the real-time control of a DC motor outlined in Figure 7.

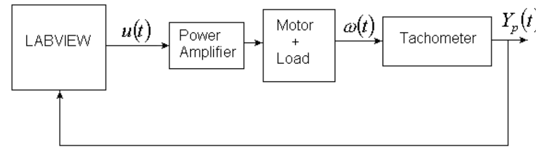

Fig. 7. DC motor speed control

Using a reaction curve an approximate motor transfer function is found to be

$$G_p(S) = \frac{17.5}{5.5S + 1}$$

the model transfer function is chosen to be

$$G_m(S) = \frac{1}{2S + 1}$$

From this the nominal values of P_0 and P_1 are

$$\hat{P}_0 = 0.157 \text{ and } \hat{P}_1 = 0.1$$

The $I\alpha T$ criterion is applied and as can be seen in Figure 8 good value of α for this application are $\alpha = 0.4$.

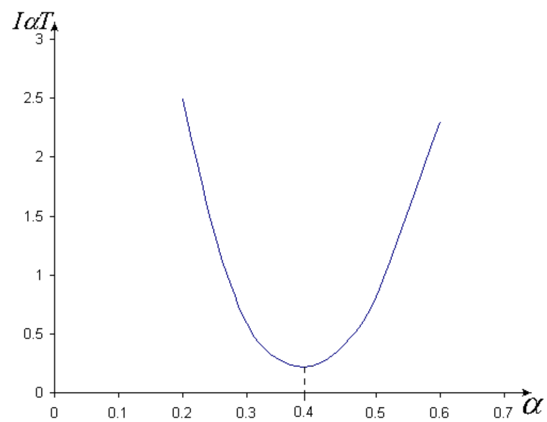

Fig. 8. Performance criterion

In Figure 9 is shown the performance of the system with $\alpha = 0.4$. From the responses the estimated values of the parameters are $P_0 = 0.14$ and $P_1 = 0.11$, approximately. From the response also it can be seen that the control system is stable, the error signal is relatively small and the parameters have settled to their steady-state values relatively quickly. This is despite the fact that the process is quite noisy and non-linear.

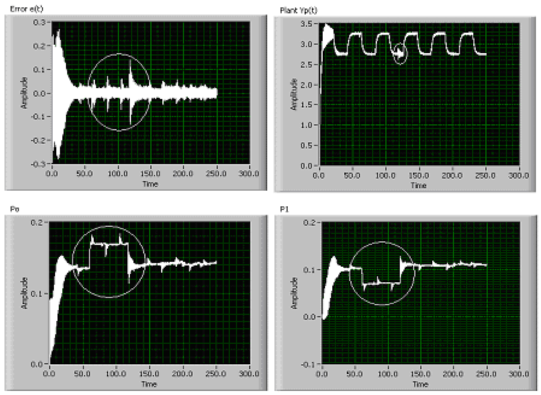

Fig. 9. DC motor response with $\alpha = 0.4$

The example also illustrates how the system responds to and recovers from a large load change.

7. CONCLUSION

This paper provides an experimental method for selecting good values of adaptation gains. It has been found to work in a wide range of simulations and real-time applications. In the absence of an analytical approach to finding optimal values of the adaptation gain it provides an easily applied method for finding good initial values for many applications including the auto-tuning of the adaptation gains (Cheung, 1996; Cerda *et al.*, 1992).

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