Design of Critical Control Systems Using Disturbance Cancellation Controllers

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Abstract: Recently, the authors have proposed a new critical control system design which does not require extensive numerical search. The key idea is to decompose the design problem into two simpler design steps by the technique used in the classical loop transfer recovery method. Since the integral action of the controller is required to deal with the rate-limited exogenous signal, our previous work assumes the use of the Davison type integral controller. In this paper, we discuss the application of the new method to the control system design using the integral controller based on the disturbance cancellation.

1. INTRODUCTION

A control problem is called critical if it requires the controlled responses of interest to remain in admissible ranges for all possible exogenous inputs. As a systematic method for designing critical control systems, Zakian (2005) has proposed the principle of matching. The design of a critical control system is reduced to find a controller satisfying a so-called matching condition given by a set of inequalities. Matching conditions have been obtained for several classes of exogenous inputs.

The priciple of the matching is usually applied for a controller with tuning parameters. The tuning parameters satisfying the matching condition are found by a numerical search method. The matching condition is usually expressed as non-convex inequality constraints on the tuning parameters. Successful use of heuristic search methods such as the moving boundary method, the genetic algorithm and the simulated annealing have been reported but these search methods cannot guarantee the success of the search. Although non-heuristic search methods guaranteeing the success of the search have been proposed, they have own difficulties and can not be applied for a broad class of the problems.

Recently, the authors (Ishihara and Ono, 2007) have proposed a new critical control system design which does not require extensive numerical search. The key idea is to decompose the design problem into two simpler design steps by the technique used in the classical loop transfer recovery method. Since the integral action of the controller is required to deal with the rate-limited exogenous signal, our previous work assumes the use of the Davison type integral controller.

In this paper, we discuss the application of the new method to the control system design using the integral controller based on the disturbance cancellation (Guo et al., 1996, Ishihara et al., 2005, Ishihara and Guo, 2008).

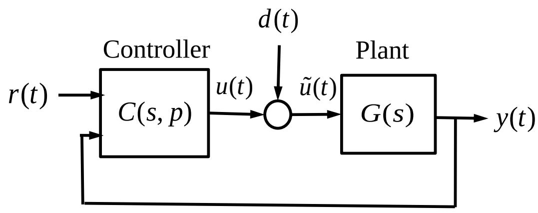

Fig. 1 Critical control system

2. PROBLEM FORMULATION

2.1 Critical Control Problem

Consider the critical control system design (Zakian, 2005) for the control system shown in Fig.1 where $G(s)$ is a scalar plant, $C(s,p)$ is a two input controller with the tuning parameter vector p, $r(t)$ is a reference input and $d(t)$ is a disturbance. Assume that the reference input $r(t)$ is a magnitude limited signal with the known bound;

$$r \in F_\infty(M) \triangleq \left\{ f : \|f\|_\infty \le M, \quad f(0)=0 \right\} \tag{1}$$

and that the disturbance signal $d(t)$ is a rate limited signal with the known rate;

$$d \in \tilde{F}_\infty(D) \triangleq \left\{ f : \|\dot{f}\|_\infty \le D, \quad f(0)=0 \right\} \tag{2}$$

The responses of interest are assumed to be the tracking error

$$e(t) \triangleq r(t) - y(t) \tag{3}$$

and the plant input

$$\tilde{u}(t) \triangleq u(t) + d(t)\,. \tag{4}$$

The design specifications are given by

$$\hat{\varepsilon}_1(p) \triangleq \sup_{r,d} \| e \|_\infty \le \varepsilon_1, \quad \hat{\varepsilon}_2(p) \triangleq \sup_{r,d} \| \tilde{u} \|_\infty \le \varepsilon_2, \tag{5}$$

where ε_1 and ε_2 are admissible bounds specified by the designer. Note that the plant input $\tilde{u}(t)$ is used in the design specification (5) rather than the control input $u(t)$. As seen in the subsequent discussions, this choice is essential for the critical control design of the rate-limited disturbance entering to the plant input side.

By the principle of matching, the specifications (5) can be replaced by the practical matching conditions:

$$\hat{\varepsilon}_1(p) = M \| g_{er}(\delta, p) \|_1 + D \| g_{ed}(h, p) \|_1 \le \varepsilon_1 \tag{6}$$

$$\hat{\varepsilon}_2(p) = M \| g_{\tilde{u}r}(\delta, p) \|_1 + D \| g_{\tilde{u}d}(h, p) \|_1 \le \varepsilon_2 \tag{7}$$

where δ is the unit impulse signal, h is the unit step signal, $g_{er}(\delta, p)$ is the unit impulse response of the transfer function from r to e and $g_{ed}(h, p)$ is the unit step response of the transfer function from d to e ; the meanings of the notations $g_{\tilde{u}r}(\delta, p)$ and $g_{\tilde{u}d}(h, p)$ are obvious.

The design of the critical control system is reduced to find a controller parameter p satisfying the inequalities (6) and (7).

2.2 Disturbance Cancellation Controller

In the standard approach for the critical control system design, the designer chooses the controller structure first and the controller parameters satisfying the matching conditions are found by a numerical search. It has been pointed out that the integral action in the controller is preferable for the rate-limited exogenous signals. We discuss the efficient design of the critical control system using an integral controller based on the disturbance cancellation technique. The design of the disturbance cancellation controllers is discussed in Guo et al. (1996), Ishihara et al. (2005) and Ishihara and Guo (2008), which is summarized as follows.

Consider a state space representation of the plant with a disturbance:

$$\dot{x}(t) = Ax(t) + B[u(t) + d(t)], \quad y(t) = Cx(t), \tag{8}$$

where $x(t)$ is a n-dimensional state vector, $u(t)$ is a scalar control input, $y(t)$ is a scalar output and $d(t)$ is a scalar disturbance.

The disturbance $d(t)$, which is assumed to be a rate-limited signal in the previous section, is temporarily assumed to be a step disturbance which obviously satisfies.

$$\dot{d}(t) = 0. \tag{9}$$

In terms of (A, B, C), the plant transfer function can be written as

$$G(s) \triangleq C(sI - A)^{-1} B \tag{10}$$

It is assumed that (A, B, C) satisfies the following assumptions:

A1: (A, B, C) is a minimal realization and $G(s)$ is non-singular for almost all s.
A2: (A, B, C) is minimum phase
A3: (A, B, C) has no zero at $s = 0$.

Augmenting the state $x(t)$ and the disturbance $d(t)$, we can construct the extended plant

$$\dot{\xi}(t) = \Phi\xi(t) + \Gamma u(t), \quad y(t) = Hx(t), \tag{11}$$

where

$$\xi(t) \triangleq \begin{bmatrix} d(t) \\ x(t) \end{bmatrix}, \quad \Phi \triangleq \begin{bmatrix} 0 & 0 \\ B & A \end{bmatrix}, \quad \Gamma \triangleq \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad H \triangleq [0 \quad C]. \tag{12}$$

It can easily be checked that the pair (H, Φ) is observable but (Φ, Γ) is not stabilizable under A1 and A3.

By the observability of (H, Φ), it is possible to construct an observer for estimating the state vector (5). A full order observer is given by

$$\dot{\hat{\xi}}(t) = \Phi\hat{\xi}(t) + \Gamma u(t) + K[y(t) - H\hat{\xi}(t)] \tag{13}$$

where

$$\hat{\xi}(t) \triangleq \begin{bmatrix} \hat{d}'(t) & \hat{x}'(t) \end{bmatrix}' \tag{14}$$

is the estimate of $\xi(t)$ and K is an observer gain matrix. The Kalman filter theory can be used to determine the observer gain matrix K by introducing a stochastic model

$$\begin{aligned} \dot{\xi}(t) &= \Phi\xi(t) + \Gamma u(t) + \bar{\Gamma} w(t), \\ y(t) &= H\xi(t) + v(t), \end{aligned} \tag{15}$$

where $v(t)$ and $w(t)$ are mutually independent zero-mean white noise processes with the covariance matrices given by $V > 0$ and $\sigma I > 0$, respectively, and the matrix $\bar{\Gamma}$ is chosen such that the pair $(\Phi, \bar{\Gamma})$ is stabilizable. Then the observer gain matrix can be determined by solving the Riccati equation which has a unique positive definite solution.

In the past works (Franklin et al. 1991, Guo et al. 1996a, 1996b, Ishihara et al. 2005), the disturbance cancellation control law has been given *a priori* without optimality consideration. It is worth noting that the disturbance cancellation is optimal for a quadratic performance as shown below.

For the extended plant (11) with the perfect observation, consider the optimal control problem with the quadratic performance index

$$J_\xi \triangleq \int_0^\infty \{y^2(t) + \rho[d(t) + u(t)]^2\} dt \tag{16}$$

where $\rho > 0$. The above performance index is quadratic but includes the cross term of the state $d(t)$ and the control input $u(t)$. The optimal control problem can easily be converted into the equivalent optimal control problem with a quadratic performance index without the cross term. Using the plant input $\tilde{u}(t)$ defined in (4), we can write the performance index (16) as

$$J_\xi = \int_0^\infty \{y^2(t) + \rho \tilde{u}^2(t)\} dt\ , \tag{17}$$

which does not include the cross term. From (4), (11) and (12), the extended plant (4) is rewritten as

$$\dot{\xi}(t) = \tilde{\Phi}\xi(t) + \Gamma\tilde{u}(t),\ \ y(t) = H\xi(t), \tag{18}$$

where

$$\tilde{\Phi} \triangleq \begin{bmatrix} 0 & 0 \\ 0 & A \end{bmatrix} \tag{19}$$

Since the disturbance $d(t)$ is eliminated from the plant dynamics in (18) as well as the performance index (17), the optimal control is given by

$$\tilde{u}(t) = -Fx(t)\ , \tag{20}$$

where F is the optimal feedback gain matrix of the standard LQ problem for the plant $\dot{x} = Ax + Bu_c$, $y = Cx$ and the performance index (17).

From (4) and (20), the optimal control for the original problem is given by

$$u(t) = -Fx(t) - d(t)\ , \tag{21}$$

which implies that the disturbance cancellation is optimal for the performance index (16) under the perfect observation of $x(t)$ and $d(t)$. Note that the optimal control (21) stabilizes the plant but not the extended plant which is obviously unstabilizable.

By the separation principle, the output feedback controller including the term for the reference input can be constructed as.

$$u(t) = -F\hat{x}(t) - \hat{d}(t) + T_F r(t)\ , \tag{22}$$

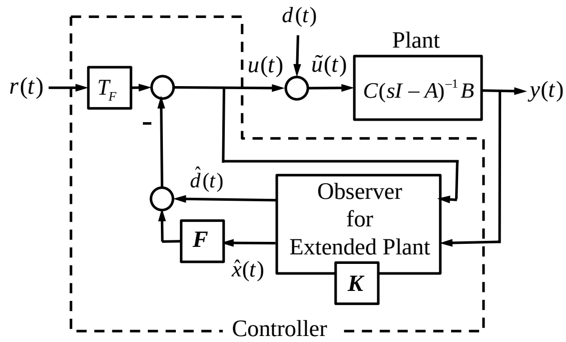

Fig. 2 Structure of the output feedback disturbance cancellation controller.

where $\hat{x}(t)$ and $\hat{d}(t)$ are the estimates of $x(t)$ and $d(t)$, respectively, generated by the observer (13) and T_F is a pre-compensation gain for the reference input.

The structure of the disturbance cancellation controller is shown in Fig. 2.

2.3 Closed Loop Expressions

Define the partition of the observer gain matrix K as

$$K = \begin{bmatrix} K_d \\ K_x \end{bmatrix}, \tag{23}$$

where K_d is a scalar and K_x is an n-dimensional vector. It can easily be checked that the Laplace transform of the estimates generated by the observer can be written as

$$\hat{x}(s) = \left[sI - A + \tilde{K}(s)C\right]^{-1}\left[Bu(s) + \tilde{K}(s)y(s)\right], \tag{24}$$

$$\begin{aligned}\hat{d}(s) = &-\frac{1}{s}K_d C\left[sI - A + \tilde{K}(s)C\right]^{-1} Bu(s) \\ &+ \frac{1}{s}K_d\left\{1 - C\left[sI - A + \tilde{K}(s)C\right]^{-1}\tilde{K}(s)\right\}y(s),\end{aligned} \tag{25}$$

where

$$\tilde{K}(s) \triangleq \frac{1}{s}BK_d + K_x\ . \tag{26}$$

It follows from (22)~(26) that the control input can be written as

$$u(s) = \begin{bmatrix} C_r(s) & C_y(s) \end{bmatrix}\begin{bmatrix} r(s) \\ y(s) \end{bmatrix} = C_r(s)r(s) + C_y(s)y(s)\ , \tag{27}$$

where

$$C_r(s) \triangleq \left[I + s^{-1}K_d C(sI - A + K_x C)^{-1} B\right] \times \left[I + F(sI - A + K_x C)^{-1} B\right]^{-1} T_F, \tag{28}$$

$$\begin{aligned} C_y(s) \triangleq & -\left[I + s^{-1}K_d C(sI - A + K_x C)^{-1} B\right] \\ & \times \left[I + F(sI - A + K_x C)^{-1} B\right]^{-1} \\ & \times F(sI - A + K_x C)^{-1} K_x \\ & - s^{-1} K_d \left[I - C(sI - A + K_x C)^{-1} K_x\right]. \end{aligned} \tag{29}$$

Note that the output can be expressed as

$$y(s) = G(s)\left[u(s) + d(s)\right]. \tag{30}$$

From (27) and (30), the transfer functions from the exogenous inputs $r(t)$ and $d(t)$ to the tracking error $e(t)$ and the plant input $\tilde{u}(t)$, with obvious notations, can be express as

$$G_{er}(s) \triangleq 1 - \left[1 - G(s)C_y(s)\right]^{-1} G(s) C_r(s), \tag{31}$$

$$G_{ed}(s) \triangleq -\left[1 - G(s)C_y(s)\right]^{-1} G(s), \tag{32}$$

$$G_{\tilde{u}r}(s) \triangleq \left[1 - C_y(s)G(s)\right]^{-1} C_r(s), \tag{33}$$

$$G_{\tilde{u}d}(s) \triangleq \left[1 - C_y(s)G(s)\right]^{-1}. \tag{34}$$

From (29) and (30), we can easily check that the term $[1 - C_y(s)G(s)]^{-1}$ can explicitly expressed as

$$\begin{aligned} \left[1 - C_y(s)G(s)\right]^{-1} = & \left[I + F(sI - A)^{-1} B\right]^{-1} \\ & \times \left[I + F(sI - A + K_x C)^{-1} B\right] \\ & \times \left[I + s^{-1} K_d C(sI - A + K_x C)^{-1} B\right]^{-1}. \end{aligned} \tag{35}$$

Note that the last term $[I + s^{-1}K_d C(sI - A + K_x C)^{-1} B]^{-1}$ has zero at $s = 0$ under the assumption A1~A3 provided $K_d \neq 0$.

Using the above results, we can easily obtain the following explicit expressions of the transfer functions.

Proposition 1: In terms of the feedback gain F, the observer gain K_x and K_d and the pre-compensator gain T_F, the transfer functions (31)~(34) can explicitly be expressed as

$$G_{er}(s) = 1 - C(sI - A + BF)^{-1} B T_F, \tag{36}$$

$$G_{ed}(s) = -C(sI - A + BF)^{-1} B \left[I + F(sI - A + K_x C)^{-1} B\right] \times \left[I + s^{-1} K_d C(sI - A + K_x C)^{-1} B\right]^{-1}, \tag{37}$$

$$G_{\tilde{u}r}(s) = [I + F(sI - A)^{-1} B]^{-1} T_F, \tag{38}$$

$$G_{\tilde{u}d}(s) = \left[I + F(sI - A)^{-1} B\right]^{-1} \left[I + F(sI - A + K_x C)^{-1} B\right] \times \left[I + s^{-1} K_d C(sI - A + K_x C)^{-1} B\right]^{-1}, \tag{39}$$

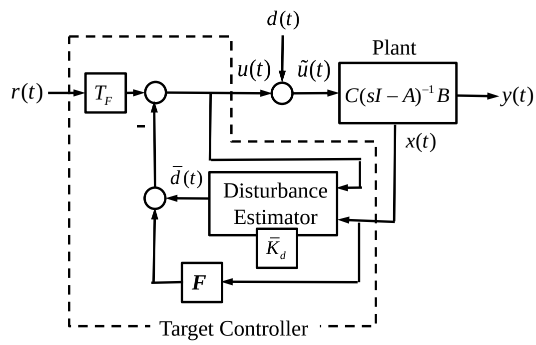

Fig. 3 The target controller with the disturbance estimator

where $G_{er}(s)$ and $G_{\tilde{u}r}(s)$ are independent of the observer gain K_x and K_d and, under the assumptions A1~A3, $G_{ed}(s)$ and $G_{\tilde{u}d}(s)$ have a zero at $s = 0$ provided $K_d \neq 0$. □

In principle, we can use the above result to find the controller parameters, if they exist, satisfying the practical matching conditions (6) and (7) by a numerical search. If we take the elements of the feedback gain matrix F and the observer gain matrix K as tuning parameters, it requires $(2n+1)$ dimensional search. Sacrificing the design freedom, we can reduce the number of the tuning parameters for F by choosing the weighting coefficient r in the performance index as a tuning parameter. To reduce the number of the tuning parameters for K, it might be possible to choose the intensity of the observation noise as the tuning parameter by fixing the covariance matrix of the system disturbance. However, this choice largely sacrifices the design freedom in choosing the gain matrix K.

Remark: The reason why we chose the plant input $\tilde{u}(t)$, rather than the control input $u(t)$, as a response of interest is explained as follows. The transfer function from the disturbance $d(t)$ to $u(t)$ is given by

$$G_{ud}(s) \triangleq \left[1 - C_y(s)G(s)\right]^{-1} C_y(s)G(s). \tag{40}$$

Unlike the transfer function $G_{\tilde{u}d}(s)$ defined in (39), the above transfer function has no zero at $s = 0$ under A3, which implies that the L_1 norm of the step response of (40) does not exist. Consequently, the choice of the control input $u(t)$ as a response of interest is inappropriate for the critical control systems design for the rate–limited disturbance entering to the plant input side.

3. TWO STEP DESIGN

To alleviate the search problem, we propose a two-step design procedure. In the first step, we determine the target state feedback controller as in the classical LTR design. The output feedback controller is obtained in the second step by the formal procedure. The tuning parameter used in this step can be interpreted as a variance of the fictitious white noise process. By simply increasing the parameter, the matching condition is asymptotically satisfied.

3.1 First Step

The target controller is constructed as follows: Assume that the plant state $x(t)$ is measurable. The plant and the disturbance model description (8) and (9) can be rearranged as

$$\dot{d}(t)=0, \quad \dot{x}(t)-Ax(t)-Bu(t)=Bd(t) \tag{41}$$

It follows from the above expression that the estimator for the disturbance state $x_d(t)$ based on the observation of the plant state $x(t)$ can be constructed as

$$\dot{\bar{d}}(t)=\bar{K}_d[\dot{x}(t)-Ax(t)-Bu(t)-B\bar{d}(t)], \tag{42}$$

where $\bar{d}(t)$ is an estimate of the disturbance $d(t)$ and $\bar{K}_d$ is an estimator gain matrix. The estimator gain matrix $\bar{K}_d$ has to be chosen such that the scalar $\bar{K}_d B$ is positive. In addition, it can easily be confirmed that the behaviour of the estimation error is determined by $\bar{K}_d B$ rather than $\bar{K}_d$.

The target controller with the reference input term is given by

$$u(t)=-Fx(t)-\bar{d}(t)+T_F r(t), \tag{43}$$

the structure of which is shown in Fig. 3.

It follows from (42) that the Laplace transform of $\bar{d}(t)$ can be written as

$$\bar{d}(s)=(s+\bar{K}_d B)^{-1}\bar{K}_d[(sI-A)x(s)-Bu(s)]. \tag{44}$$

From (42) and (43), we can write the Laplace transform of the control input as

$$u(s)=\bar{C}_r(s)r(s)+\bar{C}_x(s)x(s), \tag{45}$$

where

$$\bar{C}_r(s)=s^{-1}\left(s+\bar{K}_d B\right)T_F, \tag{46}$$

$$\bar{C}_x(s)=-s^{-1}(s+\bar{K}_d B)\left[F+(s+\bar{K}_d B)^{-1}\bar{K}_d(sI-A)\right]. \tag{47}$$

Note that

$$x(s)=(sI-A)^{-1}B[u(s)+d(s)], \quad y(s)=Cx(s). \tag{48}$$

From (46)~(48), we can easily obtain the following result.

Proposition 2: Consider the target control system shown in Fig. 3. The transfer functions from the exogenous inputs $r(t)$ and $d(t)$ to the tracking error $e(t)$ and the control input $u(t)$ can be express as

$$\begin{aligned}\bar{G}_{er}(s) &\triangleq 1-G(s)\left[1-\bar{C}_x(s)(sI-A)^{-1}B\right]^{-1}\bar{C}_r(s)\\ &=1-C(sI-A+BF)^{-1}BT_F,\end{aligned} \tag{49}$$

$$\begin{aligned}\bar{G}_{ed}(s) &\triangleq -G(s)\left[1-\bar{C}_x(s)(sI-A)^{-1}B\right]^{-1}\\ &\equiv -s(s+\bar{K}_d B)^{-1}C(sI-A+BF)^{-1}B,\end{aligned} \tag{50}$$

$$\begin{aligned}\bar{G}_{\bar{u}r}(s) &\triangleq \left[1-C_x^*(s)(sI-A)^{-1}B\right]^{-1}\bar{C}_r(s)\\ &=\left[I+F(sI-A)^{-1}B\right]^{-1}T_F,\end{aligned} \tag{51}$$

$$\begin{aligned}\bar{G}_{\bar{u}d}(s) &\triangleq \left[1-\bar{C}_x(s)(sI-A)^{-1}B\right]^{-1}\\ &=s(s+\bar{K}_d B)^{-1}\left[I+F(sI-A)^{-1}B\right]^{-1},\end{aligned} \tag{52}$$

where, under the assumption A1~A3, $\bar{G}_{ed}(s)$ and $\bar{G}_{\bar{u}d}(s)$ have a zero at $s=0$ provided $\bar{K}_d \neq 0$. □

Choosing the weighting coefficient ρ and $\lambda \triangleq \bar{K}B$ as the tuning parameters, we can consider the design specifications on the target controller

$$\bar{\varepsilon}_1(\rho,\lambda)=M\|\bar{g}_{er}(\delta,\rho)\|_1+D\|\bar{g}_{ed}(h,\rho,\lambda)\|_1\le\varepsilon_1, \tag{53}$$

$$\bar{\varepsilon}_2(\rho,\lambda)=M\|\bar{g}_{ur}(\delta,\rho)\|_1+D\|\bar{g}_{ud}(h,\rho,\lambda)\|_1\le\varepsilon_2, \tag{54}$$

which are state feedback versions of the design specifications (5) and (6), respectively. By a two-dimensional numerical search, we can find the tuning parameters ρ and λ satisfying the specifications.

3.2 Second Step

The output feedback controller satisfying the specifications (6) and (7) is obtained by the formal one-dimensional search. Let ρ^* and λ^* denote the tuning parameters determined in the first step satisfying the specifications (53) and (54).

Consider the stochastic model (15) with

$$\bar{\Gamma}=\begin{bmatrix}\lambda^*\\ B\end{bmatrix}, \tag{55}$$

which λ^* is determined in the first step.

Assume that the vaiance of the observation noise is given by $V=1$. The Kalman filter gain for the stochastic model (15) with (55) is given by

$$K(\sigma)\triangleq \Pi H'V^{-1}, \tag{56}$$

where Π is a solution of the Riccati equation

$$\Pi\Phi'+\Phi\Pi-\Pi H'V^{-1}H\Pi+\sigma\bar{\Gamma}(\bar{\Gamma})'=0. \tag{57}$$

Guo et al. (1996a, 1996b) have shown that, under the assumptions A1~A3, the pair $(\Phi,\bar{\Gamma})$ with $\lambda^*>0$ is stabilizable and (H,Φ) is detectable and that the invariant zeros of the realization $(\Phi,\bar{\Gamma},H)$ are in the open left plane. These results guarantee that the Kalman filter gain (56) satisfies the standard asymptotic property:

$$K(\sigma) = \begin{bmatrix} K_d(\sigma) \\ K_x(\sigma) \end{bmatrix} \approx \sigma^{1/2} \begin{bmatrix} \lambda^* \\ B \end{bmatrix} \tag{58}$$

Consider the output feedback controller with the feedback gain F determined by the weight ρ^* and the observer gain given by (58). The matching conditions (6) and (7) for the output feedback controller with the tuning parameter σ can be expressed as

$$\hat{\varepsilon}_1(\sigma) = M \| g_{er}(\delta,\rho^*)\|_1 + D\| g_{ed}(h,\rho^*,\lambda^*,\sigma)\|_1 \le \varepsilon_1 , \tag{59}$$

$$\hat{\varepsilon}_2(\sigma) = M \| g_{\tilde{u}r}(\delta,\rho^*)\|_1 + D\| g_{\tilde{u}r}(h,\rho^*,\lambda^*,\sigma)\|_1 \le \varepsilon_2 , \tag{60}$$

where the responses related to the reference input $r(t)$ are independent of the parameter σ and already fixed by ρ^* determined in the first step.

For the responses related to the disturbance $d(t)$, we have the following asymptotic properties.

Lemma 1: Let $G_{ed}(s,\sigma)$ and $G_{\tilde{u}d}(s,\sigma)$ denote the transfer functions defined in (37) and (39), respectively, corresponding to the feedback gain F determined by the weight ρ^* and the observer gain given by (58). Then, the transfer functions satisfy

$$\lim_{\sigma\to\infty} G_{ed}(s,\sigma) = \bar{G}_{ed}(s), \quad \lim_{\sigma\to\infty} G_{\tilde{u}d}(s,\sigma) = \bar{G}_{\tilde{u}d}(s), \tag{61}$$

where $\bar{G}_{ed}(s)$ and $\bar{G}_{\tilde{u}d}(s)$ are defined in (50) and (52), respectively.

Proof: It can easily be shown that the two matrices common in $G_{ed}(s,\sigma)$ and $G_{\tilde{u}d}(s,\sigma)$ have the following asymptotic properties:

$$\begin{aligned} &\left[sI - A + K_x(\sigma)C\right]^{-1} B \\ &\quad = (sI-A)^{-1}B\left[I + \sigma^{1/2}C(sI-A)^{-1}B\right]^{-1} \to 0 \quad (\sigma\to\infty) \end{aligned} \tag{62}$$

$$\begin{aligned} &s^{-1}K_d(\sigma)C\left[sI - A + K_m(\sigma)C\right]^{-1} B \\ &\quad = s^{-1}\sigma^{1/2}\lambda^* C(sI-A)^{-1}B\left[I + \sigma^{1/2}C(sI-A)^{-1}B\right]^{-1} \\ &\quad \to s^{-1}\lambda^* \quad (\sigma\to\infty) \end{aligned} \tag{63}$$

The asymptotic properties (61) follows by using (62) and (63) in (37) and (39) . □

Using the above lemma, we can show that a controller satisfying the matching conditions (6) and (7) is found by the two-step design.

Proposition 3: Assume that the parameters ρ^* and λ^* are determined such that the practical matching conditions (53) and (54) for the state feedback case are satisfied. Consider the output feedback controller with the feedback gain F determined by the weight ρ^* and the observer gain given by (58). Then, as the parameter σ tends infinity, the output feedback controller asymptotically satisfies the practical matching conditions (59) and (60).

Proof: Note that $g_{ed}(h,\rho^*,\lambda^*,\sigma)$ and $\bar{g}_{ed}(h,\rho^*,\lambda^*)$ are the inverse Laplace transform of $G_{ed}(s,\sigma)/s$ and that of $\bar{G}_{ed}(s)/s$, respectively. Since the both Laplace transforms are strictly proper and stable functions of s, it can be shown that $g_{ed}(h,\rho^*,\lambda^*,\sigma)$ uniformly converges to $\bar{g}_{ed}(h,\rho^*,\lambda^*)$. Similarly, we can show that $g_{\tilde{u}d}(h,\rho^*,\lambda^*,\sigma)$ uniformly converges to $\bar{g}_{\tilde{u}d}(h,\rho^*,\lambda^*)$. Therefore, the left sides of the practical matching conditions (59) and (60) satisfy

$$\lim_{\sigma\to\infty} \hat{\varepsilon}_1(\sigma) = \bar{\varepsilon}_1(\rho^*,\lambda^*), \quad \lim_{\sigma\to\infty} \hat{\varepsilon}_2(\sigma) = \bar{\varepsilon}_2(\rho^*,\lambda^*), \tag{64}$$

where $\bar{\varepsilon}_1(\rho^*,\lambda^*)$ and $\bar{\varepsilon}_2(\rho^*,\lambda^*)$ are the left sides of the practical matching conditions (53) and (54) for the state feedback case. Consequently, the matching condition (59) and (60) are asymptotically satisfied. □

4. CONCLUSIONS

The two-step method for the design of critical control systems using the disturbance cancellation controller has been proposed. The proposed method utilizes the fruits of the LQG/LTR technique to decompose the original design problem into the two steps. The parameter search required in each design step is much simpler than that required by the conventional approach. In addition, the tuning parameters have clear system-theoretic meaning, which provides the designer clear perspective. Extensions to non-minimum phase plants will be discussed elsewhere.

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