Development of Second Order Plus Time Delay (SOPTD) Model from Orthonormal Basis Filter (OBF) Model

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Abstract: A novel method to determine the parameters of a second order plus time delay (SOPTD) model from a step response is presented. The method is uniquely effective in developing SOPTD models from Orthonormal Basis Filter (OBF) model. A noise free OBF model can be easily developed from a noisy response data and any type of input with a crude estimate of time constants and no-prior knowledge of time delay. The OBF model developed in this manner can capture the dynamics of a process with only a few numbers of terms (parsimonious in parameters) and do not have the problem of inconsistency which is commonly encountered in ARX models. In addition, the OBF model gives the liberty to use any type of input sequence for identification so that we can design the best possible input sequence. However, the time delay in OBF models is estimated by a non-minimum phase zero and current methods of developing SOPTD model from a step response cannot be applied effectively. In this paper, an effective method to identify SOPTD systems or for approximating higher order systems by SOPTD model from OBF models is proposed. The efficacy of the proposed method is demonstrated through simulation studies.

Key words: - Orthonormal Basis Filter Model, System identification, Second Order Plus Time Delay model.

1. INTRODUCTION

In developing linear dynamic perturbation models for controller synthesis, Finite Impulse Response (FIR) and Auto Regressive with Exogenous Input (ARX) model structures are most commonly used in process industry. However, since these models are non-parsimonious in parameters, large data sets are required to minimize variance errors in model parameters. Orthonormal Basis Filter (OBF) models are parsimonious in parameters requiring less number of parameters. These models can be considered as a generalization of FIR models in which the filters q^{-1} , q^{-2}, … (delays) are replaced with more realistic, orthonormal basis filters, like Laguerre Filter, Kautz Filters, Markov-OBF and general orthonormal basis filters (Nelles, 2001; Patwardhan, 2005; Van Den Hof, 1995). The parameters of OBF models can be determined using least square method for both SISO and MIMO systems. Unlike ARX and ARMAX models, OBF models do not require a prior knowledge of time delay and they generally result in parameters which are consistent and unbiased. OBF models can generally be formulated either in output error or ARX structure.

System Identification using OBF models can be coupled with developing reduced complexity transfer function models like first order plus time delay (FOPTD) and second order plus time delay (SOPTD) models for two reasons. Firstly, to use OBF models for identification effectively we need a good estimate of the dominant time constants or poles of the system (Patwardhan, 2005). If the estimated pole is not good enough, the resulting OBF model needs large number of terms to capture the system dynamics (not parsimonious) and we loose one of the major advantages of OBF models. However, if identification of OBF models with development of FOPTD or SOPTD models is coupled, an iterative technique can be used with a crude initial estimate of time constant. In each iteration, a better estimate of the dominant time constant can be obtained using the transfer function model which can be used for developing more accurate and parsimonious OBF model. Secondly, there are many instances in control system design, such as PID tuning, in which it is satisfactory to obtain FOPTD or SOPTD models (Ljung, 2002). One instance is when it is intended to identify a system from closed-loop data using the controller output and the plant output as the input and output sequences, respectively, of the identification problem.

1.1 Development of OBF model

Two filters, L_m and L_n, are said to be orthonormal if they satisfy the property (Van Den Hof, 1995)

$$\langle L_m(q), L_n(q)\rangle = \begin{cases} 1 & (m = n) \\ 0 & (m \neq n) \end{cases} \tag{1}$$

where <,> represents inner product defined on the set of all stable transfer functions. Thus, a stable system, $G(q)$, can be approximately represented by a finite–length generalized Fourier series expansion as:

$$G(q) = \sum_{k=1}^{n} c_k L_k(q) \quad (2)$$

where

q = shift operator

c = the model parameters

$L_k(q)$= the orthonormal basis filters for the system $G(q)$

In time domain, the response, $y(k)$, of an LTI system for an input, $u(k)$, can be described as

$$\begin{aligned} y(k) &= c_1 L_1(q)u(k) + c_2 L_2(q)u(k) + ... \\ &+ c_m L_m(q)u(k) + v(k) \end{aligned} \quad (3)$$

where $v(k)$ describes the white noise.

The first step in the OBF model development is the selection of an appropriate type of orthonormal basis filter. The selection is based on a prior knowledge of the behaviour of the process, i.e., whether the process is well damped or not. The various types of Orthonormal Basis Filters are discussed below:

Laguerre Filters

The Laguerre filters are first-order lag filters with one real pole. They are, therefore, more appropriate for well damped processes (Nelles, 2001).

$$L_n = \sqrt{(1-p^2)}\frac{(1-pq)^{n-1}}{(q-p)^n}, \quad |p| < 1 \quad (4)$$

where p = pole (estimated)

If an estimate of the time constant t is available the corresponding pole can be calculated by:

$$p = \exp(-t / T_s) \quad (5)$$

where T_s is the sampling interval.

If a good estimate of the time constant is not available, an iterative technique can be employed in which a crude estimate of a time constant is used as a starting point and better estimates are obtained from the OBF model thus developed. A detailed discussion is available in Lemma *et al*., 2007.

Kautz Filter

Kautz filters allow the incorporation of a pair of conjugate complex poles; they are therefore effective for modelling weakly damped processes (Nelles, 2001). The Kautz filters are defined by

$$L_{2n-1} = \frac{\sqrt{(1-a^2)(1-b^2)}}{q^2 + a(b-1)q - b} f(a,b,q) \qquad n = 1, 2, \ldots \quad (6a)$$

$$L_{2n} = \frac{\sqrt{(1-b^2)}(q-a)}{q^2 + a(b-1)q - b} f(a,b,q) \qquad n = 1, 2, \ldots \quad (6b)$$

where

$$f(a,b,q) = \left(\frac{-bq^2 + a(b-1)q + 1}{q^2 + a(b-1) - b} \right)^{n-1} \quad (7)$$

$$a = \frac{2Re}{1 + Re^2 + Im^2} \quad (8)$$

$$b = -(Im^2 + Re^2) \quad (9)$$

$$|a| < 1, \ |b| < 1$$

Re = real part of the pole

Im = imaginary part of the pole

Markov-OBF

When a system involves time delay and an estimate of the time delay is available, Markov-OBF can be used. The time delay in Markov-OBF is included by placing some of the poles at the origin.

Van Den Hof, *et al.*, (1995) discussed Generalized Orthonormal Basis Filters and its relation with the other orthonormal basis functions in detail.

To estimate the model parameters, the regression matrix, X, is formulated first. It is formed by filtering the inputs with the appropriate orthonormal basis filters (Nelles, 2001).

$$X = \begin{bmatrix} u_{L1}(m) & u_{L2}(m-1) & \ldots & u_{Lm}(1) \\ u_{L1}(m+1) & u_{L2}(m) & \ldots & u_{Lm}(2) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ u_{L1}(N-1) & u_{L2}(N-2) & & u_{Lm}(N-m) \end{bmatrix} \quad (10)$$

where $u_{Li} = Li(q,p)u(k)$ are the inputs filtered with the appropriate orthonormal basis filters. The model parameters are then calculated by the linear least square formula:

$$c_k = (X^T X)^{-1} X^T y \quad (11)$$

1.2 SOPTD model

Once an OBF model is developed, a step change in its input is introduced and a noise free response is obtained. The OBF model developed in this way approximates the time delay by a non-minimum phase zero and this approximation appears as an inverse response in the step response of the OBF model. Figure 1 depicts a typical step response of an OBF model. Because of the approximate nature of the time delay, current methods of developing a SOPTD model from step response of OBF models are not effective. The two commonly used methods are discussed below.

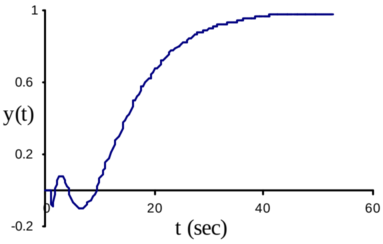

Fig.1. A typical step response of an OBF model

The Smith method is a graphical method of determining the SOPTD parameters from the step response of a system (Smith, 1972). The time at which the normalized step response reaches 20% (t_{20}) and 60% (t_{60}), with apparent time delay removed, are first determined from the step response data. The value of the damping coefficient, z, and t/t_{60} are then determined from a graph using the ratio t_{20}/t_{60}. From t/t_{60} and t_{60}, the natural period, t, is calculated. The method can be used to identify both underdamped and overdamped systems.

Rangaiah and Krishnaswamy (1994, 1996) proposed various methods for determining the parameters SOPTD models. For underdamped systems they presented two different methods. The methods are based on finding three points that minimize the integral absolute error (IAE) between the actual response and the step response of a SOPTD model by which the process is to be approximated. Seborg, *et al*, (2004) indicates that the methods works quite well for the range $0.707 \leq z \leq 3.0$.

The Smith technique has major difficulties in its application. Removing the apparent time delay in finding t_{60} and t_{20} is not a simple task and it is even more difficult for OBF step responses. Graphical method for estimating the apparent time delay is usually inaccurate and the parameter estimation is seriously affected. The Rangaiah and Krishnaswamy method gives good results only in a limited range and, in addition, it doesn't treat both the underdamped and overdamped cases together.

2. PRESENT WORK

In this work, a novel method for determining the parameters of the SOPTD model is presented. The method is uniquely effective in developing SOPTD models from Orthonormal Basis Filter (OBF) models. It can be used to identify both underdamped and overdamped second order systems with or without apparent time delay or to approximate a higher order system with a SOPTD model. It eliminates the need of estimating the apparent time delay separately and enables to determine all the parameters including the apparent time delay with high accuracy.

The transfer function of a second order system with time delay is given by (12):

$$G(s) = \frac{Y(s)}{U(s)} = \frac{Ke^{-t_d s}}{t^2 s^2 + 2zts + 1} \qquad (12)$$

where

K = steady state gain
t_d = time delay
z = damping coefficient
t = natural period of oscillation

To estimate the parameters, a step input is introduced into the OBF model and the step response is determined. The steady state gain can then be estimated by finding the ultimate response and dividing it by the step size, A.

$$K = \frac{y_\infty}{A} \qquad (13)$$

Once the steady state gain is estimated the normalized response is obtained by dividing the original response by the steady state gain. The inflection point of the normalized response curve is the point at which the tangent to the curve attains the maximum slope. It can be easily found by determining the instant of maximum slope, *i.e.*, by filtering the normalized response $y(k)$ with the filter given by (14) and determining the value of k that maximizes Δy .

$$\Delta y = \left(1 - q^{-1}\right) y(k) \qquad (14)$$

It is found, analytically, that the damping coefficient depends only on the value of the normalized response at the inflection point. The dependence is shown in Fig. 2. Therefore, by determining the normalized response at the inflection point the damping coefficient can be obtained from the figure.

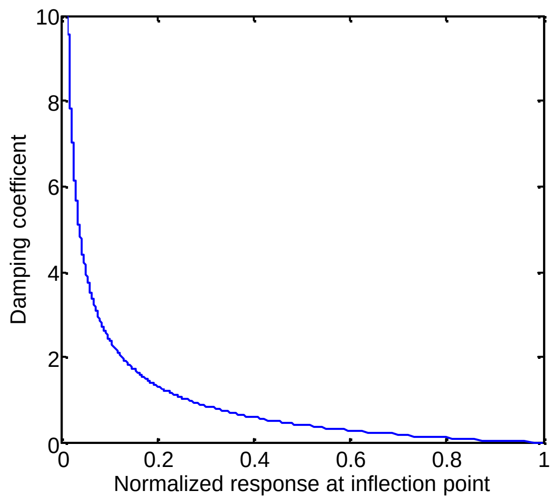

Fig. 2. Damping coefficient as a function of the normalized response at the inflection point

The most commonly used method for determining the time delay is the maximum slope method. In this method, a tangent is drawn at the inflection point and the intersection point to the time axis is taken as the approximate value of the time delay. However, this approximation for second order and higher order systems is not accurate enough. In the present work, the time delay by the maximum slope method is divided into two parts: the apparent time delay and the contributed time delay (the tail of the sigmoid curve).

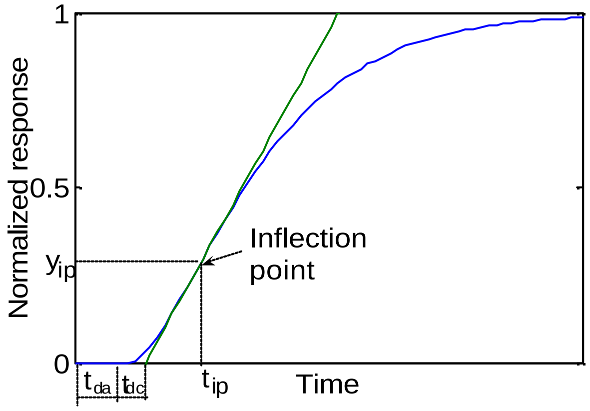

Fig. 3. The apparent and contributed time delay.

The total time delay (t_d) determined by the maximum-slope method is the sum of the apparent time delay (t_{da}) and the contributed time delay t_{dc}. Hence, the apparent time delay can be calculated by subtracting the contributed time delay from the time delay determined by the maximum slope method. The contributed time delay, t_{dc}, does not depend on the pure time delay therefore it can be calculated from the parameters of the second order transfer function without the apparent time delay.

$$t_{da} = t_d - t_{dc} \tag{15}$$

The total time delay is estimated by (16) if the response time t_{ip}, slope s_{ip} and normalized response y_{ip} all at the inflection point are determined as noted before.

$$t_d = t_{ip} - \frac{y_{ip}}{s_{ip}} \tag{16}$$

It was found that the contributed time delay and the natural period t are directly proportional to any difference of response time, t_n - t_m, and the coefficients of proportionality depend only on the damping coefficient and the specific values of t_m and t_n chosen. In this paper, t_m and t_n are chosen as the time to reach 30% and 40% of the maximum response, respectively. Since we use time differences and not absolute time, all the calculations do not depend on the apparent time delay, and hence, unlike the Smith method it is not required to estimate the apparent time delay separately. The apparent time delay itself is calculated without any difficulty using (15), (16) and (17).

$$t_{dc} = m_1(t_{40} - t_{30}) \tag{17}$$

$$t = m_2(t_{40} - t_{30}) \tag{18}$$

The dependence of m_1 and m_2 on the damping coefficient is as presented in Fig. 4.

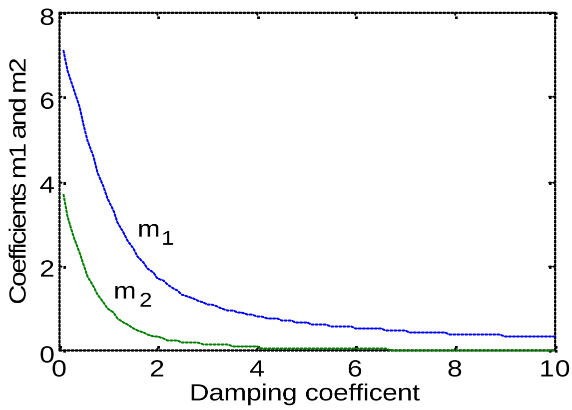

Fig. 4. m_1 and m_2 as functions of the damping coefficient

To generate the curves in Fig. 4, introduce a step input into any known second order model with various values of the damping coefficient, z, determine the corresponding t_{30} and t_{40} values for each step response and calculate the corresponding m_1 and m_2 values using (17) and (18).

3. SIMULATION STUDY

In this study, the proposed method is used for both identifying a second order plus time delay system and approximating a higher order system by a SOPTD model from a noisy response data. In the first case, data from a closed loop system and in the second case data from open loop system are used. Both data sets are generated using SIMULINK.

3.1 Identification of a SOPTD system

The data for identification is generated using the closed loop system shown in Fig. 5. The transfer function of the process is given by (19).

$$G_1(s) = \frac{2e^{-4s}}{60s^2 + 16s + 1} \tag{19}$$

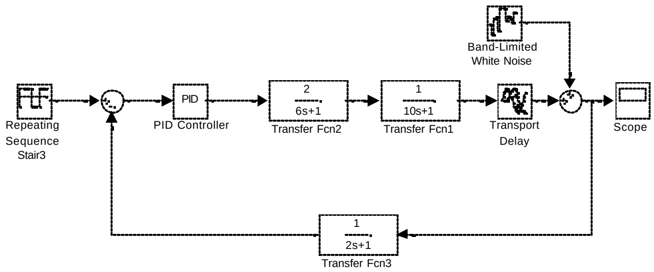

Fig. 5. The closed-loop block diagram for generating the identification data using SIMULINK.

An additive white noise with a signal to noise ratio (SNR) of 5.83 is added . SNR is defined as the ratio of the variance of the input signal to the variance of the noise signal. Proportional-only controller with proportional controller gain, K_c=1, is used. Set point changes as shown in Fig. 6 are introduced to the system.

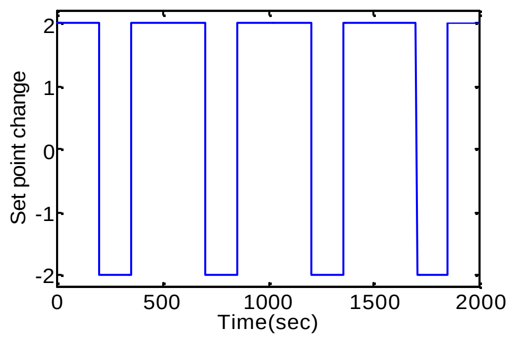

Fig. 6. Input changes in set point.

The controller and the plant outputs for the set point changes are shown in Fig.7.

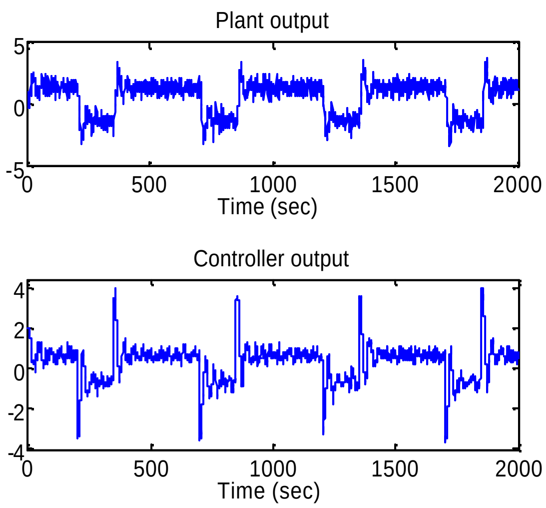

Fig. 7. Controller and plant output data from the closed loop system.

An OBF model with eight Laguerre filter terms and a single pole is developed iteratively starting with an initial time constant of 30 s (p = 0.9672, and T_s = 1s) using the controller output and the plant output as input-output data of the system identification problem. The value of the pole, p, in the Laguerre filter converges to 0.8826 in four iterations with the corresponding model parameters, c_k , equal to [0.1571, 0.2974, 0.05664, -0.0330, 0.0306, 0.0124, 0.0007].

The SOPTD model developed from the OBF model is given by (20).

$$G_{1m}(s) = \frac{2.019e^{-4.09}}{64.25s^2 + 16.57s + 1} \tag{20}$$

Fig. 8 shows the step response of the actual SOPTD system and the approximate SOPTD model identified from the noisy plant output and controller output data.

OBF models with output error structure developed from closed-loop data theoretically leads to biased parameter estimates.

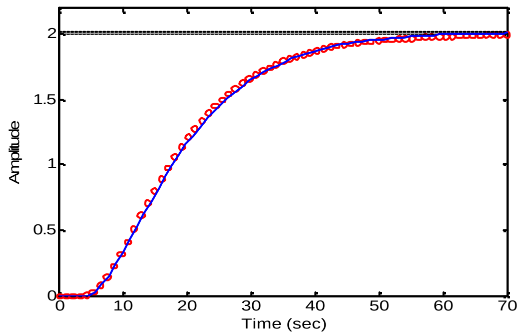

Fig. 8. Comparison of step responses of the original (ooo) and the identified (——) systems.

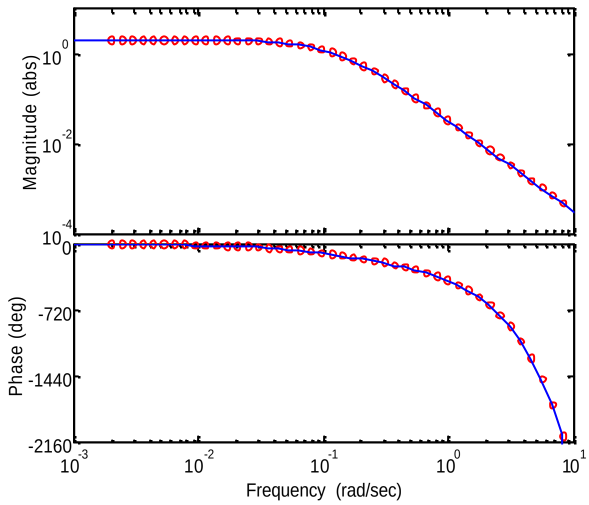

Fig. 9. Bode plot of the original (ooo) and the identified SOPTD (——)models.

However, for the simulation example, it is observed that the bias in the estimates does not affect the result significantly. Even though there is some difference between the actual and the estimated parameters, the step responses and the Bode plots (Fig. 9) show insignificant differences especially for control system design.

3.2 Approximation of a fourth order system by SOPTD model

In this example, a fourth-order system with a time delay given by (21) is considered. The open-loop response of the system with additive white noise, SNR=5.08, for a Pseudo Random Binary Sequence (PRBS) input is shown in Fig. 10.

$$G_2(s) = \frac{6.5e^{-6s}}{28.8s^4 + 68.4s^3 + 50.5s^2 + 13.3s + 1} \tag{21}$$

An input-output data is generated for the system using SIMULINK. An OBF model with eight Laguerre filters is developed with an initial time constant of 40 s (p = 0.9753, and T_s = 1s).

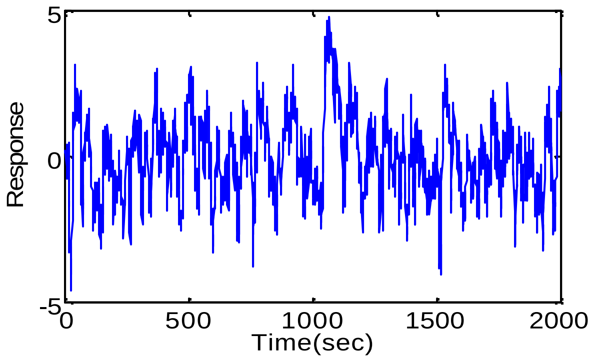

Fig. 10. The response of the fourth order model with additive white noise (SNR=5.08) for PRBS input

In four iterations the pole, p, in the Laguerre filters converges to 0.8601 with the model parameter estimate, c_k, equal to [0.3207, 1.0145, 0.6787, -0.3044, -0.0071, 0.1434, 0.1587, 0.0985]. A SOPTD model is obtained from the step response of the OBF model and the resulting transfer function is given by (22). The step responses of the original fourth order system and the SOPTD approximation are presented in Fig. 11. It is observed that the step response of the SOPTD model and that of the original fourth order system are very close.

$$G_{2m}(s) = \frac{6.512e^{-7.97s}}{35.93s^2 + 11.63s + 1} \tag{22}$$

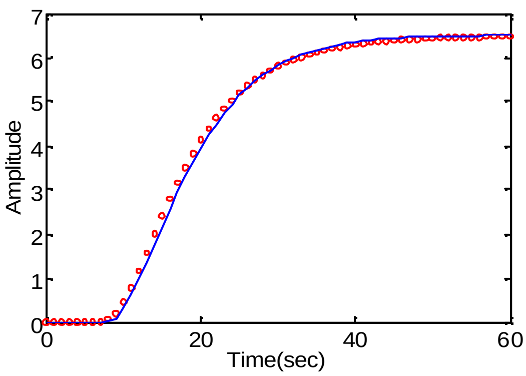

Fig. 11. The step responses of the original (ooo) and the approximate SOPTD models (——)

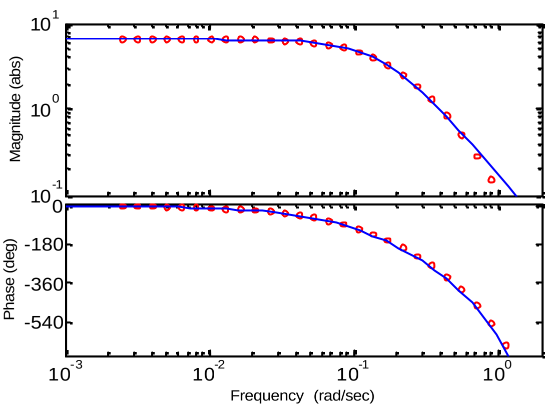

Fig.12. Bode diagram of the original fourth order system (ooo) and the approximate SOPTD model (——).

The bode diagram of the original fourth order and it's approximation by SOPTD model is presented in Fig. 12. It can be noted from the bode plot that the approximate SOPTD model gives good result in the frequency range useful for control system design.

4. CONCLUDING REMARKS

A novel method for estimating the parameters of an overdamped SOPTD model from OBF model is presented. Models for underdamped systems can be estimated using either generalised OBFs or Gautz filters. It is shown that the parameters of a SOPTD model can be effectively estimated without the need of estimating the apparent time delay separately. The approximation of higher order systems by SOPTD model is shown to provide accurate results both in time and frequency domains.

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