Optimal Mode Decomposition for high dimensional systems

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Introduction

- Want to find low-order representations of high dimensional systems to implement into estimation and control

- OMD - algorithm which provides linear low order approximation of dynamical systems (data driven methodology)

- Conjugate gradient algorithm with an input of data snapshots outputs relevant mode shapes with corresponding linear dynamics

- Objectives:
  - *Estimate the performance in the variety of applications and compare against similar methods*
  - *Establish link between the method and concepts from linear operator theory*
Example – fluid flow system

Cylinder flow (Re = 60) – limit cycle

- System governed by nonlinear Navier Stokes equations
- Limit cycle characterised by periodic vortex shedding
- Complex system can be approximated with a sum of oscillating structures

Eigenvalue lattice

- Data snapshots cover limit cycle and final part of the transition
- Asymptotic eigenvalues represent oscillatory behaviour
- Values with negative growth rate – settling on limit cycle
- Each eigenvalue has a corresponding structure
Examples of Mode Shapes

\[ \lambda_0 = 0 \]

\[ \lambda_1 = 0 \pm 0.866i \]

\[ \lambda_2 = 0 \pm 1.732i \]
OMD - algorithm

- Set up data in a series of snapshots
  \[ A = \{x_1, x_2, ..., x_N\} \quad \text{B} = \{x_1^+, x_2^+, ..., x_N^+\} \quad A, B \in \mathbb{R}^p \times N \]

- Solve the optimisation problem
  \[
  \min_{M, L} \|B - LML^T A\|^2 \quad \text{s.t.} \quad L^T L = I \\
  L \in \mathbb{R}^{p \times r} \quad M \in \mathbb{R}^{r \times r} \quad r < p
  \]

- Optimal low-order linear matrix – \( M^*(L) \)
  \[
  \frac{\partial \|A - LML^T B\|^2}{\partial M} = 0 \\
  M^*(L) = L^T A B^T L (L^T B B^T L)^{-1}
  \]

- Use iterative methods to find optimal \( L \)

- Dynamics described with eigenvalues of \( M \) and corresponding modes
  \[
  \Phi = LP \quad \text{where} \quad M = P \Delta P^{-1}
  \]
Some comparisons with established techniques

Identification of the relevant structures

OMD method

Established technique

Noise filtering

Reference values
OMD
Other technique

OMD - magnitude difference
Other technique - magnitude difference
## Conclusion or summary or future work

### Conclusions:
- Iterative algorithm which estimates the low-rank approximation of the high-dimensional system was presented

### Future work:
- Further explore performance of OMD and compare it with other methods
- Explore links and attempt to create better approximations of theoretical concepts from linear operator theory
- Combine OMD with algorithms which attempt to recover ‘missing data’