Supercapcitor Design Using Systems Theory

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What are supercapacitors?

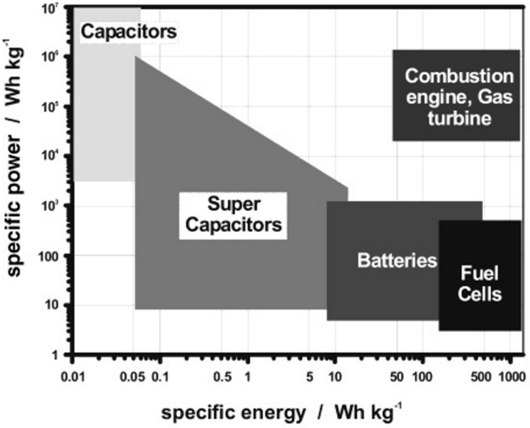

Figure: Energy Vs. Power per unit mass

Figure: Supercapacitor

Why are we interested in them?

Figure: Growth in number of publications with "supercapacitor" keyword.

Electrochemical based model

- Charge conservation across the double layer

$$aC\frac{\partial(\phi_1 - \phi_2)}{\partial t} = \sigma\frac{\partial^2\phi_1}{\partial x^2} \quad (1)$$

- Electrolyte diffusion

$$\epsilon\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} - \frac{aC}{F}\left(t_-\frac{dq_+}{dq} + t_+\frac{dq_-}{dq}\right)\frac{\partial(\phi_1 - \phi_2)}{\partial t}, \quad (2)$$

- Ohm's Law

$$\kappa\left(\frac{RT(t_+ - t_-)}{F}\right)\frac{\partial}{\partial x}\ln\,(c) + \sigma\frac{\partial(\phi_1 - \phi_2)}{\partial x} + \left(\kappa\frac{\partial}{\partial x} + \sigma\frac{\partial}{\partial x}\right)\phi_2 + i = 0 \quad (3)$$

Local absolute stability with sector and slope restrictions

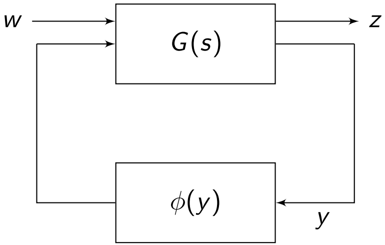

Figure: Lur'e System.

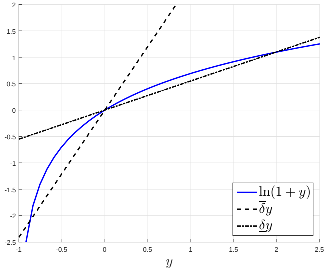

Figure: Logarithmic nonlinearity.

$$V(x) = \begin{bmatrix} x & \phi(y) \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12}' & P_{22} \end{bmatrix} \begin{bmatrix} x \\ \phi(y) \end{bmatrix} + \sum_{i=1}^{m} \lambda_i \int_0^{y_i} \phi(\sigma) - \underline{\delta}\sigma \; d\sigma \quad (4)$$