Relay feedback based monitoring and autotuning of processes with gain nonlinearity


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Abstract: Performance assessment and monitoring of control systems can be used to improve the performance of industrial processes. In this paper, a novel relay feedback based method for monitoring and autotuning of a class of proportional-integral (PI) controllers is proposed for the systems with gain nonlinearity. For performance assessment of the closed loop system, a time domain evaluation criteria based on the integral of the absolute value of the error (IAE) and the normalized pick of the error in setpoint (SP) changes are presented. Simulation results on the highly nonlinear pH process have shown the effectiveness and feasibility of this method.

Keywords: Relay feedback; Autotuning; Performance monitoring; pH control process;

1. INTRODUCTION

Ill-performed control systems not only greatly reduce system effectiveness, but also lead to low efficiency, unqualified products and cost. Therefore, process engineers have to retune the parameters of controller whenever the process dynamics have changed due to nonlinearity, disturbances, aging or equipment changes as well as initial process operation. So many control engineers have recognized the importance of monitoring and automatic tuning of the controllers and the problem is widely studied for the past two decades (Yu, 2006).

Usual performance indicators in the time domain with reference to the step response are rise time, settling time, overshoot (OS), IAE, or the integral of the square of the error (ISE). In the frequency domain, some typical performance indicators are bandwidth, amplitude gain margin, phase gain margin, resonance module or sensitivity bandwidth.

After introducing the relay feedback test by Astrom and Hagglund (1984), it has received much attention of process control practitioners and many researches have been done to extend the application of relay feedback in monitoring and retuning loops as well as critical point or transfer function (TF) estimation and autotuning. Chiang et al. (1993) proposed a frequency domain monitoring procedure based on the relay feedback for single input/ single output (SISO) systems. Ju and Chiu (1997) further extended their work to multi-loop control systems. However, in both the above studies the relay experiment has to be carried out at least twice for online evaluation of the maximum closed loop log modulus. Thyagarajan et al. (2003) introduced the shape factor into controller monitoring to devise a simpler procedure. A relay feedback approach for controller tuning based on assessment of gain and phase margin proposed by Jeng et al. (2006) and Balestrino and Landi (2006) used an IAE index based on a period of output data under relay feedback test.

Beyond monitoring, to recognize the inadequacy of the controller performance, it is important to ascertain whether an acceptable level of performance can be achieved with the exiting control structure. If it is possible, then the desirable performance is attainable by retuning the current controller. Some performance assessment methods give suitable information for developing retuned controllers, or other tuning tests will be carried out to retune the controller (Balestrino and Landi, 2006). The important point is choosing the tuning test and its compatibility with both controller structure and process specifications like stability or gain nonlinearity. Relay feedback based method is a good alternative because of its effectiveness and simplicity.

The relay test is a closed loop test. Therefore, by choosing proper relay parameters, the output of the process will not drift away from the nominal operating point. Regarding to this advantage, relay feedback tests are scheduled to identify the local linear models in different operating points of nonlinear system (Jeng et al., 2005, and Lin et al., 2006). In these cases, the relay feedback test has been performed in some operating points for systems with static nonlinearity.
and a controller is tuned in each setpoint with respect to the obtained information. However, in practice, a systematic method is needed to evaluate the estimated local linear model and its peer controller to recognize when a new relay test should be started. Also it is more time efficient and convenient that the existing controller be kept in closed loop, under the relay autotuning test.

To attain these goals, in this paper a novel Monitoring relay (M-relay) feedback test is proposed for automatic retuning the internal model control (IMC) based PI controller for processes with static gain nonlinearity. In this method at the first operating point, with relay feedback test a local linear first order plus dead time (FOPDT) model is fitted to the plant and the primary PI controller will be tuned. After that, when SP changes, the performance criteria based on IAE and OS, evaluate the performance of nonlinear system and compare it with the performance of the estimated local linear model. If the criteria are not satisfied, then the M-relay feedback test will be carried out in presence of both PI controller and process. By using the output data of M-relay, the parameters of local linear model and PI controller will be retuned. In the operating points between two tuned points, the gain of linear model and controller can be calculated easily from a linear interpolation of tuned setpoints and related gains.

This paper is organized as follows. Some concepts of conventional relay feedback and TF estimation are presented in the section 2. In the section 3 the proposed Monitoring relay (M-relay) is introduced and evaluation criteria are defined to make the M-relay run automatically. In the section 4 a complete monitoring and autotuning test based on M-relay will be carried out on the simulated pH nonlinear model to illustrate the effectiveness of the procedure. Conclusions are drawn in the section 5.

2. THE RELAY FEEDBACK TEST AND TF IDENTIFICATION

As mentioned in the section 1, the relay feedback procedure is an effective yet simple technique for parameter estimation that has become adopted in the industrial process controllers. The relay experiment was basically performed to derive the ultimate gain and ultimate frequency parameters, used in Ziegler-Nichols tuning algorithm. Since process models in terms of transfer functions play an important role in process analysis and control, several attempts have been recorded in the literature about deriving TF from relay tests.

Many of the industrial processes especially chemical ones with small static nonlinear gain can be modelled by FOPDT transfer function. According to this fact, in this paper we concentrate on this type of processes and in this section a summary of conventional relay feedback structure and FOPDT model approximation will be reviewed. The parameters of the PI controller, presented in the next section, are calculated based on the estimated FOPDT model.

2.1 Relay feedback and approximate transfer function modelling

Fig. 1 shows the main structure of the relay feedback autotuning where \( y \) is the controlled output, \( e \) is the error and \( u \) is the manipulated input. A relay with hysteresis band is placed in the feedback loop. The Astrom-Hagglund relay feedback test is based on the observation, when the output lags behind the input by \( -\pi \) radians, the closed loop system may oscillate with a period of \( P_u \). Fig. 2 illustrates how the relay feedback system works. The period of the limit cycle is the ultimate period and from the Fourier series expansion, the amplitude \( a \) can be considered to be the result of the primary harmonic of the relay output. Therefore, the ultimate frequency and ultimate gain can be approximated as following:

\[
\omega_u = \frac{2\pi}{P_u}
\]

(1)

\[
K_u = \frac{4h}{\pi a}
\]

(2)

![Fig. 1. The conventional relay feedback system](image)

![Fig. 2. The limit cycle oscillation for a system with positive steady state gain](image)
\[ G_c(s) = \frac{K_c e^{-D \tau_s}}{\tau_s + 1} \]  
(3)

The identification of the FOPDT systems consist of three steps. At the first step, the time to the peak amplitude should be recorded as \( D \) besides \( a, h, P \). In the second one, the time constant \( \tau \) is calculated from (5) in an iterative manner and (4) provides an initial guess for \( \tau \). Finally the gain \( K_p \) is computed from (6).

\[ \tau = \frac{\tan(\pi - D \hat{\theta}_i)}{\hat{\theta}_i} \]  
(4)

\[ \tau = \frac{\pi}{\omega_n \ln(2e^{\omega_n^2} - 1)} \]  
(5)

\[ K_r = \frac{a}{h(1-e^{-\omega_n\tau})} \]  
(6)

2.2 IMC based PI controller

In a conventional feedback control system comprising a FOPDT process \( G_p \) in series with a PI controller \( G_c \), the controller and the loop transfer function \( G_{ol} \) have the general forms of (7) and (8).

\[ G_c(s) = \frac{K_c(1 + \tau_s)}{\tau_s} \]  
(7)

\[ G_{ol}(s) = G_r G_c(s) = \frac{K_r K_c(\tau_s + 1)e^{-\tau_s}}{\tau_s(\tau_s + 1)} \]  
(8)

If the integral time \( \tau_i \) is selected equal to the constant time \( \tau \) of the process, it is converted to the original IMC-PI tuning rule (Rivera et al., 1986). With this integral time, the loop transfer function has a special structure of the integrator plus dead time (9)

\[ G_{ol}(s) = \frac{K_c K_r e^{-\tau_s}}{\tau_s} = \frac{e^{-\tau_s}}{\tau_s} \]  
(9)

\[ \varepsilon = \frac{\tau_i}{K_r K_c} \]  
(10)

This model based tuning rule has a good performance in the SP tracking by choosing the proper gain, but slower damping for disturbance rejection and it is straightforward. In the proposed M-relay the open loop integrator structure of this tuning method is used.

3. PERFORMANCE ASSESSMENT AND AUTOMATIC CONTROLLER RETUNING

The framework for the online performance assessment and the controller tuning is illustrated in Fig. 3, where the \( M \) indicates switching to M-relay for the nonlinear gain identification and automatic retuning of the IMC-PI controller and \( P \) indicates performance evaluation mode of the regular feedback system.

At the first stage of the operation, an IMC-PI controller is tuned based on the derived local linear FOPDT model of the plant, estimated from the conventional relay feedback in the section 2.1.

![Fig. 3. Relay feedback monitoring and autotuning system](image-url)

To verify the controller performance against the other operating points, an evaluation criteria in the time domain is presented, computed from the data collected during the transient output response of the process in the changes of setpoint. If the criteria exceed the acceptable threshold, an online closed loop relay test in the presence of both controller and the nonlinear process, is proposed for retuning the local linear FOPDT model and respectively the IMC-PI controller, without substantial modifications of the operating point.

3.1 Proposed monitoring relay (M-relay) structure

In each SP changes, the performance is assessed and the controller should be retuned with running the M-relay, if the performance is not satisfied. In the proposed M-relay structure (Fig. 3) a hysteresis relay has to be put in the loop before the controller and the system, to monitor a control system with the conventional feedback structure. Whenever the system switches to \( M \), the relay runs. The step response of an integrator \( G_{ol} \) is a ramp function and that of a pure dead time process is just a step delay in the time. Hence, the relay output response of the loop transfer function is triangular in shape. The half period of the relay response corresponds to the dead time \( D \) and its slope corresponds to the \( D/\varepsilon \), according to the Fig. 4 and (11).

![Fig. 4. Output response of GOL (solid line) and its input (dash line)](image-url)

The important factor is the \( \varepsilon \), which gives the value of the gain of the PI controller and affects the output performance. If the optimal value of \( \varepsilon \) becomes defined, the gain of the primary PI controller can be obtained from (10). Also, in retuning procedure of the PI controller with \( \tau_i = \tau \) and under the M-relay test with amplitude \( d \), the last value of the \( \varepsilon \) \( (\varepsilon_{old}) \) is computed from (11), according to Fig. 4 and afterwards the gain of the retuned PI controller will be updated from (12), in order to adjust the new value of the \( \varepsilon \) \( (\varepsilon_{new}) \) to its optimal range.
\[
\varepsilon_{\text{old}} = \frac{Dd}{a} \quad (11)
\]
\[
K_{\text{new}} = \left( K_{\text{old}} \right)(\varepsilon_{\text{new}})/(\varepsilon_{\text{new}}) \quad (12)
\]

The value of the \( \varepsilon \) should be optimized based on a performance criterion. The IAE or ISE cost functions are proper alternatives for this purpose. Several tests have been carried out on the FOPDT systems, controlled with the mentioned IMC-PI controller. In each test the IAE and ISE is calculated in the SP tracking for the same system with different values of the \( \varepsilon \). From Fig. 5, it is observed that the \( \varepsilon/D=1.73 \) results in the minimum of IAE in the SP tracking and the optimal performance of the closed loop system under the PI controller with \( \tau_{\text{new}}=\tau \).

![Fig. 5](image)

Fig. 5. Effect of \( \varepsilon/D \) on IAE (solid) and ISE (dashed) for FOPDT processes with \( \tau_{\text{new}}=\tau \), using PI controller

According to Fig. 5, the optimal value for the \( \varepsilon/D \) is 1.73 with minimizing the IAE cost function and it is 1.37 for the ISE cost function. But the closed loop performance with the IAE index is smoother and it is more desirable in nonlinear systems, to avoid large OS. So \( \varepsilon/D =1.73 \) will be considered in all the related formulas to retune the gain of the PI controller:

\[
\varepsilon_{\text{new}} = 1.73D \quad (13)
\]

In fact, by this method the linear PI controller is generalized for a type of nonlinear processes and the controller is able to automatically retune itself versus the gain nonlinearities, originated from the structure of the plant or disturbances. The nonlinear curve of the process gain, obtained in the retuning procedure, is an important specification especially in chemical processes. For example in the pH neutralization process, the approximated titration curve can be achieved from the nonlinear curve of the estimated gain.

In the next section, several simulations on the highly nonlinear pH process model, illustrate the effectiveness and the simplicity of the proposed method for the automatic relay monitoring test to retune the PI controller and identification of a system with highly nonlinear gain.

### 3.2 Performance evaluation criteria

Since the PI controller is tuned based on the IMC rule, for a FOPDT estimated system in a particular operating point, the main objective of this performance criteria is assessment and comparison of the real process output and the estimated FOPDT local linear model in series with the same PI controller, in the changes of the SP. As mentioned earlier, an IAE performance index (14) is used. The index is calculated from the collected data in the period of change in the SP \( t_{\text{SP}} \), till the 6 times of the constant time \( \tau \) for both the linear model and real process outputs. In the period of \( t=6\tau \), the step output of the FOPDT linear system is equal to the output in the time of \( t/2 \) with less than 5 percent error:

\[
J_{\text{IAE}} = \frac{1}{t_{\text{SP}}} \int_{0}^{t_{\text{SP}}} e(t)dt \quad (14)
\]

\[
\frac{y(6\tau) - y(3\tau)}{y(6\tau)} = 0.05 \quad (15)
\]

Relative \( IAE = J_{\text{IAE}}/J_{\text{IAE-linear}} \quad (16) \)

To define a proper relation for the IAE indices of the linear estimated model (\( J_{\text{IAE-linear}} \)) and the real process (\( J_{\text{IAE}} \)), the ratio of these indices (Relative \( IAE \)) is plotted against the variations of the nonlinear gain of the real process to estimate the gain of its linear model. If the relative \( IAE \) index becomes less than 1, it means a better performance in the real process output. Therefore, there is no need for retuning the controller. But if the index exceeds some threshold larger than 1, it means deficiency in the performance of the real system and maybe it has damping oscillations with large OS or smooth and slow performance with big rise time. According to Fig. 6, the relative ratio of 1.25 is selected for this criterion. Fig. 7 shows the performance of the nonlinear process versus increasing/decreasing variations of the nonlinear gain with the same ratio of 1.25 for the relative \( IAE \) index.

![Fig. 6](image)

Fig. 6. Effect of the nonlinear gain variations on the relative \( IAE \) index

Fig. 7 shows that when the nonlinear gain is in decreasing mode (A), the output slows down and the 1.25 is a critical value for the relative \( IAE \). By reducing this factor, the settling time of the controlled process response becomes longer than the linear model ones, and it is not an acceptable performance. But in increasing mode of the nonlinear gain (B), the relative \( IAE \) can not be the only criterion for performance assessment. Other specifications should also be considered. Here, due to the nonlinear gain and its effect on amplifying the OS when the process gain is in increasing mode, we augment the normalized pick amplitude of the step response to the relative \( IAE \) performance condition. The amount of OS criterion can be modified by the operator based on the specifications of each process. For existing pH model, the maximum OS of 20% is defined.
Finally, the performance assessment criteria are defined as follows:

\[
\begin{align*}
\text{Relative } IAE & \leq 1.25 \\
O.S. &= 100 \times \frac{\Delta y_{\text{me}} - SP}{\Delta SP} \leq 20\% 
\end{align*}
\] (17)

Fig. 7. Comparison of the nonlinear system performance with the linear model with the relative \( IAE = 1.25 \), (A) decreasing and (B) increasing mode of the nonlinear gain

If they are not satisfied, then the feedback system in Fig. 3, switches to \( M_i \), to retuning the controller. If the SP changes all over the operating points, with \( M_{-\text{relay}} \) monitoring and retuning procedure, the linear PI controller is generalized in all operating points of the process with gain nonlinearity. The controller is able to automatically retune itself versus the gain nonlinearities, originated from the structure of the plant or disturbances. Moreover, the gain of the process is estimated in each run of the \( M_{-\text{relay}} \) as follows:

\[
K_{r_{\text{new}}} = \frac{\tau_i}{K_{r_{\text{new}}} r_{\text{new}}}
\] (18)

Using this relation, the nonlinear curve of the process gain can be obtained, which is an important specification especially in the chemical processes. For example, in the pH neutralization process, the approximated titration curve can be achieved from the curve of the estimated gain.

4. SIMULATION TESTS ON pH PROCESS

In this section, several simulations on a highly nonlinear pH model is performed to illustrate the effectiveness and the simplicity of the proposed method in monitoring and automatic retuning the PI controller and identifying the nonlinear gain property, with the \( M_{-\text{relay}} \) test.

4.1 The nonlinear pH process

The pH control process has a continuous stirred-tank reactor (CSTR) with three input streams: sodium hydroxide (NaOH), acetic acid (CH\(_3\)COOH) and the water. The base flow is used to control the pH of the CSTR and the water flow controls the level of the tank. The acid has a constant flow and is considered as the input disturbance. The initial conditions of the process is on \( \text{pH}_{\text{base}} = 12 \), \( \text{pH}_{\text{acid}} = 3 \), the controlled level of the tank \( (h) = 10 \) (cm). The standard deviation of the measurement noise is 0.01. All the three input signals are based on 0-100% full range (10 liter per hour). The flow of the acid is 15%.

Fig. 8. The pH neutralization system

The nonlinear curve of the gain variations for the pH system with the mentioned specifications is shown in Fig. 9.

Fig. 9. Nonlinear gain of the pH process

4.2 Monitoring and tuning procedure

At the first stage of the test, the pH of the process is set to 5 with manual step changes in base flow and then a conventional relay test is carried out, to estimate the primary local linear FOPDT model of the process (19). Fig. 10 clearly indicates the FOPDT type of performance of the pH process.

\[
G_{\text{pH}}(s) = \frac{0.24e^{-380s}}{380s + 1}
\] (19)

Fig. 10. Start-up and relay test for the primary FOPDT model estimation and the PI controller tuning

According to Fig. 9, the gain of the pH process changes from 0.2 in \( \text{pH} = 5 \) up to 41 in \( \text{pH} = 8 \) and again decreases to less than 0.1 in \( \text{pH} = 11 \). So if the parameters of the tuned controller in \( \text{pH} = 5 \) remain fixed, the output does not perform well in the other setpoints and may become unstable. Simulations show the closed loop instability for the current controller, starts from \( \text{pH} = 6.1 \).
To avoid this problem, after tuning the first controller, the performance evaluation criteria will assess the output performance in the SP tracking and if it is necessary, the feedback control system will be switched to the M-relay unit to retune the PI controller and to estimate the new gain of the process. To provide a good coverage of the SP range (pH=5-11), the SP is increased with 0.2 amplitude in each level. Fig 11 is the results of the complete procedure, but only a part of the full range is shown to avoid the lack of clarity.

Fig. 11. Performance monitoring and M-relay autotuning test

Fig. 12 shows the nonlinear gain estimation based on the proposed closed loop M-relay and the open loop step method. The amplitude of the SP changes in monitoring test is selected to be 0.3. Smaller changes can yield better coverage for the operating points and more accurate approximation of the nonlinear gain. However, it takes more runs of M-relay test and therefore a longer time to complete the test.

Fig. 12. Approximated and real nonlinear gain for pH process

After monitoring and autotuning based on the proposed M-relay feedback, the retuned values of the gain of the PI controller are stored. Therefore, in other points, the gain $K_C$ is calculated by a linear interpolation between the two related gains. The tracking test in Fig. 13 has an acceptable performance in all the operating points of pH, especially for small changes of SP. But larger changes in SP result in some deviations from optimal performance. This problem can be solved by taking account of the other parameters like the SP in computing the controller gain in each operating point and a set of the fuzzy rules is a good alternative to this aim.

Fig. 13. SP tracking under the IMC-PI controller, based on the estimated nonlinear gain

5. CONCLUSIONS

As demonstrated in this paper, the performance monitoring approach helps the operator to visualize whether the ultimate performance is achieved from the experiment and if the output performance is not acceptable, with the proposed method based on the monitoring relay feedback test, the parameters of the controller will be updated.

This method is adapted especially for the processes with static gain nonlinearity and the FOPDT dynamic. To implement an automatically retuned PI controller on a highly nonlinear system, the simulation studies on a pH process are provided.

REFERENCES


